

# EC9C5 Topics in Labour Economics Assignment 1

## Frictional Markets and Estimation

**Instructions:** The due date is 8 January 2025. Turn in on Tabula in one single pdf. You should include all script files you used.

- 1.) **The Model:** Consider a world where workers receive a flow value of leisure  $b$ . For now let's consider the contract rate of job offers to be fixed at  $\lambda$ . When a firm and a worker match they both observe a match specific productivity draw  $\theta$  from an exogenous distribution  $G(\theta)$  with support  $[0, \infty)$ , that is output from a job is equal to  $\theta$ . After observing the match specific productivity the wage is determined by Nash Bargaining in which the worker has a bargaining power of  $\beta$ . Existing matches exogenously break up at rate  $\delta$ . Workers and firms discount the future at rate  $r$ .
  - a. Write down the value functions ( $V_u$  and  $V_e$ ), for the worker and the expected discounted value of a filled match.
  - b. Solve for the Nash Bargained wage and the reservation wage as a function of the value of unemployment (you will not be able to solve for the reservation wage as a function of the primitive parameters). Are these a function of the match specific productivity? Why or why not? Also solve for the reservation match productivity, i.e., the productivity value that leads to the reservation wage, denote it  $\theta^*$ .
  - c. Solve for the accepted wage distribution in terms of the productivity distribution. What is the support of the accepted wage distribution?
  - d. Solve for the steady state unemployment and employment rate.
- 2.) **The Policy:** Now let's impose a minimum wage into the model. A minimum wage  $m$  is the lowest possible wage that can be paid. Denote  $V_u(m)$  as the new value of unemployment with a minimum wage  $m$ .
  - a. A minimum wage is considered to be binding if workers would be willing to accept jobs at wages below the minimum wage. In the model, when is a minimum wage binding?
  - b. Firms are not willing to have negative profits, with a binding minimum wage, which matches, i.e. which productivity levels, are no longer feasible?
  - c. Solve for the productivity level for which the Nash Bargained wage is exactly equal to the minimum wage, call it  $\hat{\theta}$ . When is  $\hat{\theta} > m$ ?
  - d. For a binding minimum wage, are matches that have a productivity  $\theta \in [m, \hat{\theta}]$  feasible? If so what is the wage at these matches be? What is the wage for matches with  $\theta > \hat{\theta}$ ? Solve for the accepted wage distribution. What is the support of the expected wage distribution?
  - e. Write down the new value of unemployment  $V_u(m)$  for a binding minimum wage.
  - f. Solve for the steady state unemployment and employment rate for a binding minimum wage.

- 3.) **The Likelihood Function:** The data you have contains observations of worker outcomes under two different minimum wages. You can observe if a worker is unemployed or employed, what their wage is if they are employed, and the unemployment duration of unemployed workers.
- The unemployed's contribution:* Derive the joint probability of observing an unemployed worker with duration  $t$ . Assume that the arrival rate of job offers is Poisson.
  - The employed's contribution:* Derive the joint probability of observing an employed worker at wage  $w > m$  and the joint probability of observing an employed worker working at the minimum wage.
  - Write down the likelihood of observing the data at both minimum wages allowing the arrival rate and reservation wage at each minimum wage to differ. Denote these values  $\lambda_0$ ,  $\lambda_1$  and  $w_0^r$ ,  $w_1^r$ .
- 4.) **The Estimation:** For this part you will have to assume that the workers bargaining power is  $\beta = 0.5$  and invariant to the minimum wage. To estimate the bargaining power you would need matched worker-firm data. Assume that the productivity distribution is log-normal with mean  $\mu$  and standard deviation  $\sigma$ .
- Maximize the likelihood function allowing for different arrival rates and reservation wage for each minimum wage. Report the point estimates, standard errors, and likelihood value.
  - Maximize the likelihood function allowing for a different reservation wage for each minimum wage. Report point estimates, standard errors, and the likelihood value.
  - Maximize the likelihood function allowing for a different arrival rate for each minimum wage. Report point estimates, standard errors, and the likelihood value.
  - Using the likelihood values from parts a. - c., test if the contact rate and reservation wage change when the minimum wage changes.
  - Explain why we can identify the reservation wage and the arrival rate in this model.
  - If you were to use SMM to estimate the model, list the moments that you would need and explain how they identify the model parameters.