

The Job Arrival Rate and DMP

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From last time

$$rU = b + \alpha \int_{w_R}^{\infty} E(w) - U dF(w)$$

$$rE(w) = w + \delta[U - E(w)]$$

- We now have some ways of thinking about $F(w)$
- What about α ?
 - typical assumption: poisson arrival rate
 - what does it represent?
 - what does this arrival rate depend on?

The arrival rate of jobs

- At the beginning we assumed you get a job offer every period
 - with an exogenous wage distributions we have unemployment only because you received a job offer less than your reservation wage
- Now let's assume there is an arrival rate of jobs
 - frictional unemployment: unemployed because you did not get an offer
 - why did you not get an offer: coordination frictions

Coordination Frictions

- Trade in the labor market is decentralized
 - firms make decisions about how many jobs to create and wages to offer
 - workers make decisions about where to apply to job, how many jobs to apply to, ect ...
- More than 1 worker applies to the same job: **unemployment**
- No worker applies to a certain job: **unfilled vacancy**
- Burdett, Shi, Wright (2001)

Burdett, Shi, Wright

- Environment
 - Two workers (1 and 2) homogeneous and looking for work, each apply to only one job
 - Two firms (A and B) homogeneous and each have one job to fill
 - If the job is filled output = y , and wage = w
 - One shot game
- Payoffs
 - If a match occurs

$$u = w \quad \pi = y - w$$

- If no match occurs

$$u = 0 \quad \pi = 0$$

Burdett, Shi, Wright

- Firms choose a wage to offer
 - w_A and w_B
- Workers choose which job to apply to
 - worker i applies to firm A with prob = θ_i
 - worker i applies to firm B with prob = $1 - \theta_i$
- Two stage game
 - Stage 1: Firms post wages
 - Stage 2: Workers choose probabilities

Stage 2

- Worker takes wages as given.
- Worker 1's utility from applying to firm A and firm B

$$U_{1A} = \frac{1}{2}\theta_2 w_A + (1 - \theta_2)w_A$$

$$U_{1B} = \theta_2 w_B + \frac{1}{2}(1 - \theta_2)w_B$$

- Worker 2's utilities are symmetric
- Worker 1 is indifferent between applying to both jobs ($U_{1A} = U_{1B}$) if

$$\theta(w_A, w_B) = \frac{2w_A - w_B}{w_A + w_B}$$

Stage 2

- Worker 1's strategy

$$\theta_1 \begin{cases} 0 & \text{if } \theta_2 > \theta(w_A, w_B) \\ 1 & \text{if } \theta_2 < \theta(w_A, w_B) \\ [0, 1] & \text{if } \theta_2 = \theta(w_A, w_B) \end{cases}$$

- Worker 2's strategy is symmetric
- When does $\theta(w_A, w_B) = 1$?

$$w_A > 2w_B$$

- When does $\theta(w_A, w_B) = 0$?

$$w_A < \frac{1}{2}w_B$$

- When is $0 < \theta(w_A, w_B) < 1$?

$$\frac{1}{2}w_B < w_A < 2w_B$$

Stage 2

- If $w_A > 2w_B$, both workers are better off going to firm A

$$\theta_1 = \theta_2 = 1$$

- If $w_A < \frac{1}{2}w_B$, both workers are better off going to firm B

$$\theta_1 = \theta_2 = 0$$

- If $\frac{1}{2}w_B < w_A < 2w_B$ there are three equilibria

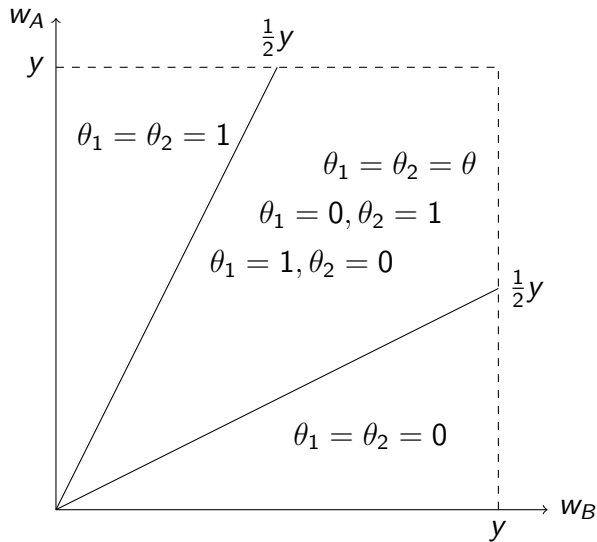
- Pure strategy: (θ_1, θ_2) is $(0, 1)$ or $(1, 0)$

- Perfect coordination

- Mixed strategy: $\theta_1 = \theta_2 = \theta(w_A, w_B)$

- Coordination frictions

Stage 2



Stage 1

- Taking the workers strategies as given solve for firm profits
 - If $w_A > \frac{1}{2}w_B$ then firm A gets both workers

$$\pi_A = y - w_A, \pi_B = 0$$

- If $w_A < 2w_B$ then firm B gets both workers

$$\pi_A = 0, \pi_B = y - w_B$$

- If $\frac{1}{2}w_B < w_A < 2w_B$, in both pure strategy equilibria

$$\pi_A = y - w_A, \pi_B = y - w_B$$

Stage 1

- If it posts $\frac{1}{2}w_B < w_A < 2w_B$, and workers play a mixed strategy, Firm A's profits are:

$$\begin{aligned}\pi_A &= (y - w_A)\theta_1(1 - \theta_2) + (y - w_A)(1 - \theta_1)\theta_2 \\ &\quad + (y - w_A)\theta_1\theta_2 + 0(1 - \theta_1)(1 - \theta_2) \\ \pi_A &= (y - w_A)\frac{3w_B(2w_A - w_B)}{(w_A + w_B)^2}\end{aligned}$$

- Firm A's profit maxing best response:

$$w_A^*(w_B) = \frac{w_B(4y + w_B)}{5w_B + 2y}$$

- Firm B's profits and best response are symmetric

Mixed Strategy Equilibrium

- Solving for the equilibrium

$$\theta_1 = \frac{1}{2}$$

$$w_A = \frac{y}{2}$$

$$\pi_A = \frac{3y}{8}$$

$$\theta_2 = \frac{1}{2}$$

$$w_B = \frac{y}{2}$$

$$\pi_B = \frac{3y}{8}$$

Mixed Strategy Equilibrium

- The expected number of matches

$$M = 1\theta_1\theta_2 + 1(1 - \theta_1)(1 - \theta_2) + 2(1 - \theta_1)(\theta_2) + 2\theta_1(1 - \theta_2)$$

- Expected probability of receiving job offer (assuming that the firm randomly chooses between the two workers if both apply to the same job)

$$\alpha = \frac{M}{2} = 0.75$$

General Solution

- Burdett, Shi, Wright show that for m firms and n workers the matching function is:

$$M(m, n) = m \left[1 - \left(1 - \frac{1}{m} \right)^n \right]$$

- Arrival rate of job offers $M(m, n)/n$
- Fix $n/m = b$ then as m increases the arrival rate decreases \Rightarrow matching function is decreasing returns to scale. Bigger markets have larger frictions
- As $m \rightarrow \infty$ matching function converges to constant returns to scale

The Matching Function

- Typically in labor search models we do not explicitly model the application strategies of workers.
 - Reduced form approach to matching friction: assume a matching function exists
 - Matching function:
 - depends on the number of unemployed U and vacancies V
 - depends on some aggregate efficiency parameter A
 - exhibits constant returns to scale
- $$M(U, V) = AU^\beta V^{1-\beta}$$
- Nice discussion: Petrongolo & Pissarides (2001)

The Job Finding and Filling Rate

- Given the matching function

$$M(U, V) = AU^\beta V^{1-\beta}$$

define labor market tightness $\theta = V/U$

- The job finding rate

$$p(\theta) = \frac{M(U, V)}{U} = A\theta^{1-\beta}$$

- The job filling rate

$$q(\theta) = \frac{M(U, V)}{V} = A\theta^{-\beta}$$

Diamond Mortensen Pissarides (DMP)

- Environment
 - continuous time
 - everyone discounts at rate r
 - homogeneous workers searching for jobs
 - homogeneous firms post vacancies
 - job finding and filling rates determined by matching function
 - wages determined by Nash Bargaining

Diamond Mortensen Pissarides (DMP)

- A steady state
 - a measure of unemployed workers u
 - a measure of vacancies v
 - a wage w
- We have three unknowns so we will have three steady state equations to solve
 - (1) **The Beveridge Curve**: a relationship between the unemployment rate and vacancy rate
 - (2) **Job Creation**: firms continue to post vacancies until the value of having a vacant job is zero
 - (3) **The Nash solution**: gives a solution to the wage as a function of labor market tightness
- (2) and (3) will determine the steady state values of θ^* and w^* . Given θ^* , (1) and (3) will determine the steady state values of u^* and v^*

Workers

- When unemployed, workers receive unemployment benefits b and search for jobs
- The value of unemployment is

$$rU = b + p(\theta)[E - U]$$

- When employed workers receive wage w and lose their jobs at an exogenous rate δ
- The value of employment is

$$rE = w + \delta[U - E]$$

Workers

- Solving the value of unemployment and value of employment simultaneously gives:

$$E = \frac{w(r + p(\theta)) + \delta}{r(r + p(\theta)) + \delta}$$

$$U = \frac{(r + \delta)b + p(\theta)w}{r(r + p(\theta)) + \delta}$$

Beveridge Curve

- In steady state the inflow and outflow of unemployment are equal
 - Inflow: $\delta(1 - u)$
 - Outflow: $p(\theta)u$
- Solving for u gives the unemployment rate

$$u = \frac{\delta}{\delta + p(\theta)}$$

- Since $\theta = v/u$, where v is the vacancy rate, this gives us a relationship between u and v known as the Beveridge Curve.

Firms

- If a firm has a vacant job it pays flow cost κ to post the vacancy
- The value of having a vacancy is

$$rV = -\kappa + q(\theta)[J - V]$$

- If a firm has a filled job it produces output y and pays wage w , the job is exogenously destroyed at rate δ
- The value of a filled job is

$$rJ = y - w + \delta[V - J]$$

Job Creation Curve

- The free entry condition means that firms will continue to post vacancies until the value of a vacancy is driven to zero. This implies:

$$V = 0$$

$$J = \frac{y - w}{r + \delta} \quad \& \quad J = \frac{\kappa}{q(\theta)}$$

- Equating the two values for a filled job gives us the second equation we need

$$y - w - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

Wages

- Wages are determined by bargaining between the firm and the worker to avoid the critiques raised by Rothchild and Diamond
- The bargaining problem
 - Total value of a match

$$\Omega = E(w) + J(w)$$

- Disagreement values: (U, V)
 - Bargaining power: γ
- Generalized Nash Bargaining problem

$$w = \underset{w}{\operatorname{argmax}} [E(w) - U]^\gamma [J(w)]^{1-\gamma}$$

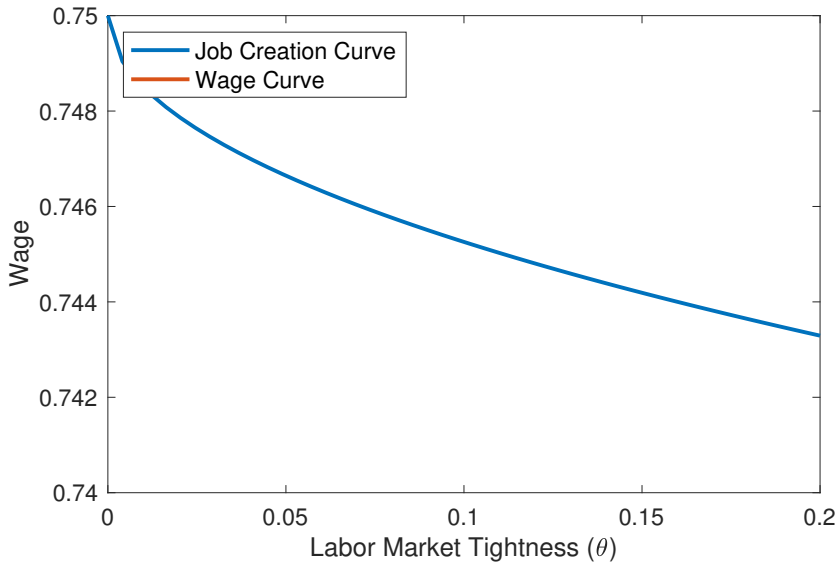
Wages

- Plugging in the value functions and solving for the max gives us the last equation we need to find the steady state

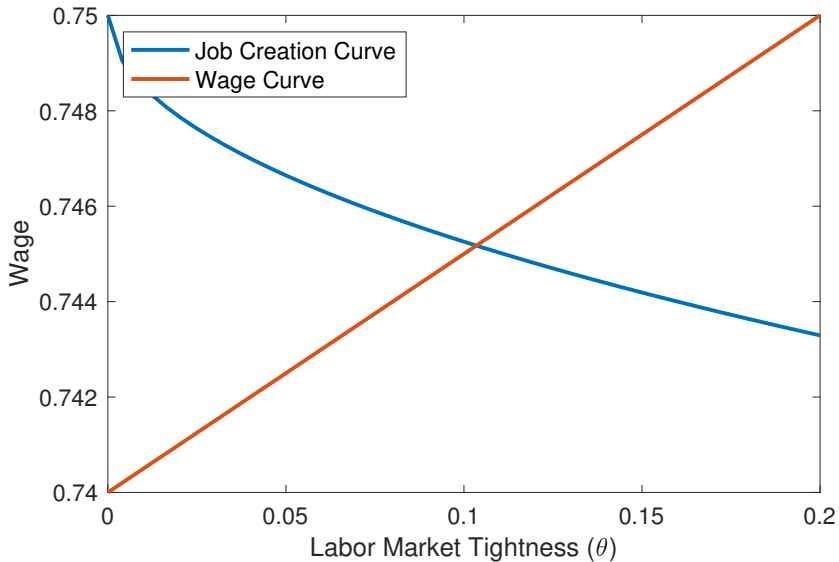
$$w = \gamma y + (1 - \gamma)rU$$

- Use FOC, the value function for rU , and $J = \kappa/q(\theta)$ to get:

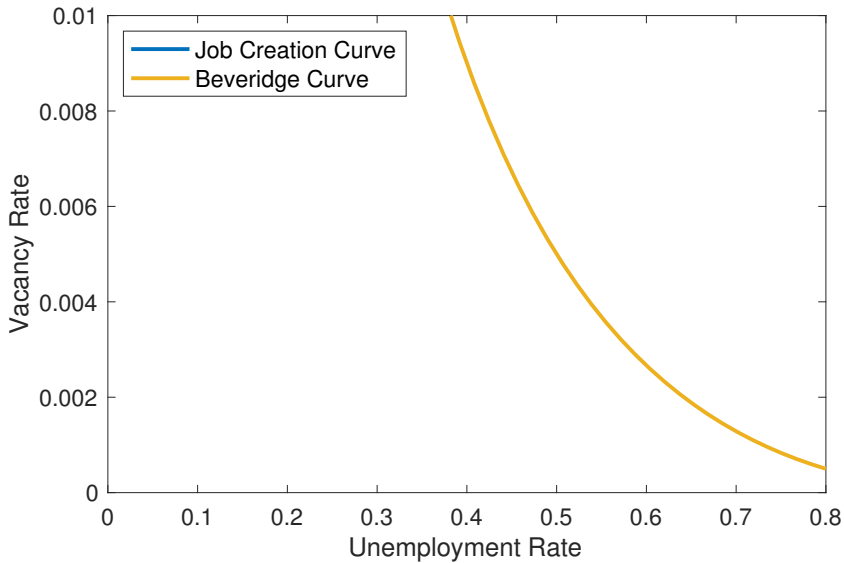
$$w = (1 - \gamma)b + \gamma(y + \kappa\theta)$$



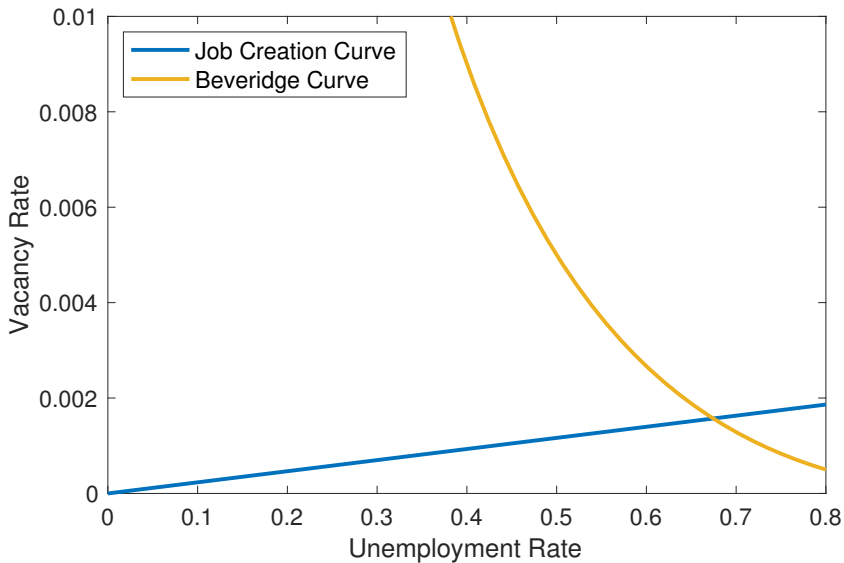
$$V = 0 \Rightarrow J = \frac{y - w}{r + \delta} \quad \& \quad J = \frac{\kappa}{q(\theta)}$$



$$w = (1 - \gamma)b + \gamma(y + \kappa\theta)$$



$$u = \frac{\delta}{\delta + p(v/u)}$$



$$v/u = \theta^*$$

Comparative Statics

- What will happen if we decrease the worker's bargaining power (γ)?
 - Job Creation?

$$y - w - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

- Wages?

$$w = (1 - \gamma)b + \gamma(y + \kappa\theta)$$

- Beveridge Curve?

$$u = \frac{\delta}{\delta + p(\theta)}$$

- Steady state?

Comparative Statics

- What will happen if we decrease the worker's bargaining power (γ)?

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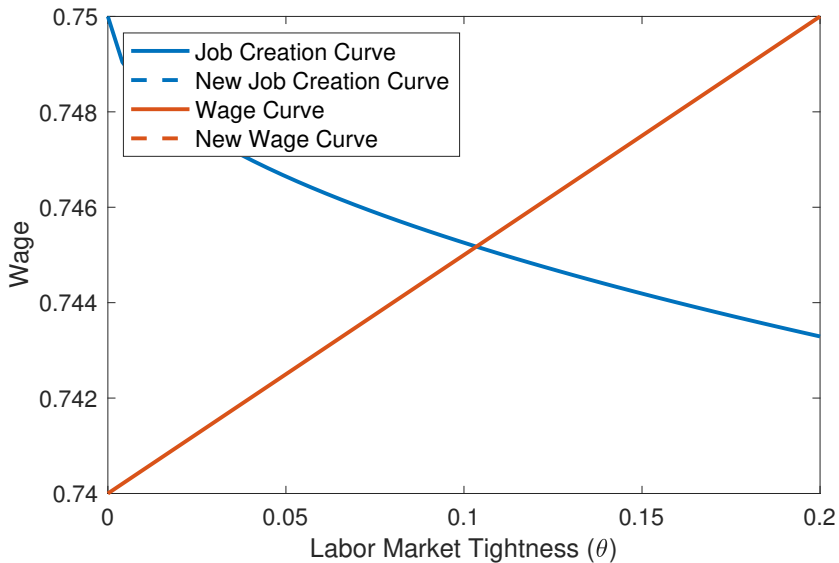
- Wages?

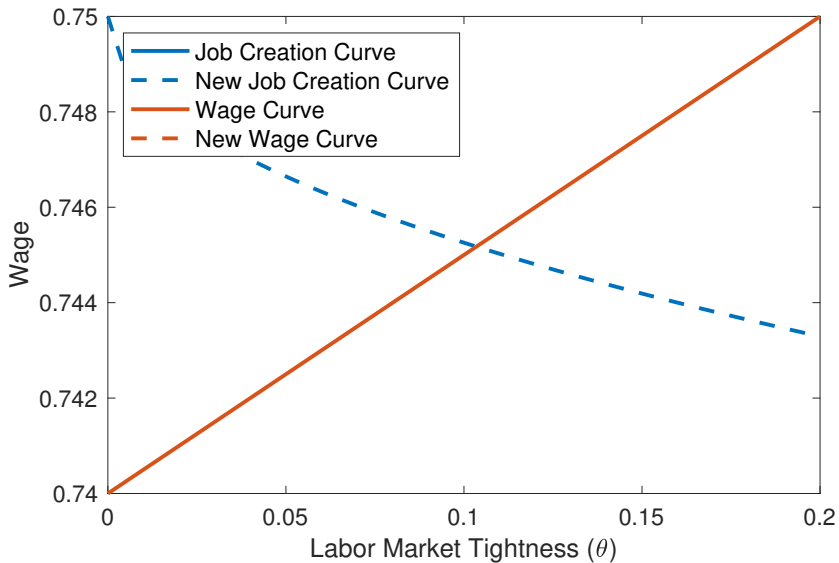
$$w = (1 - \gamma)b + \gamma(y + \kappa\theta)$$

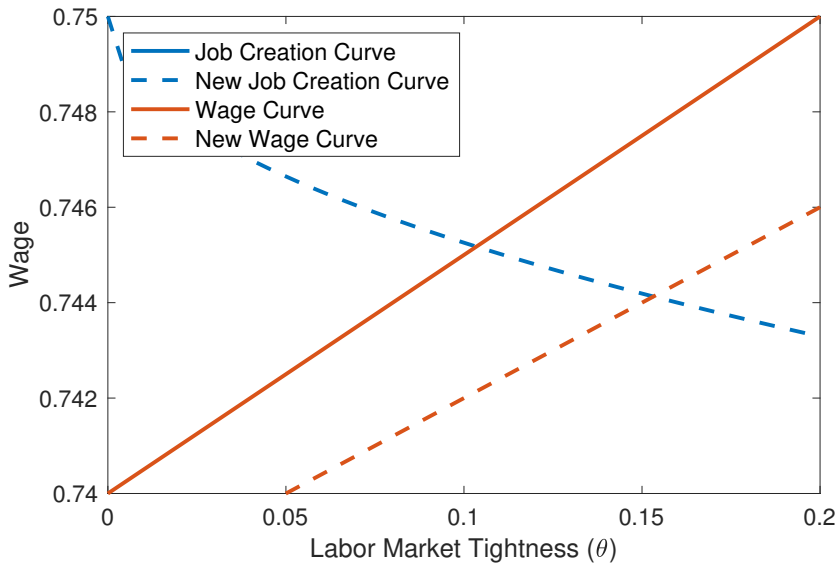
- Beveridge Curve?

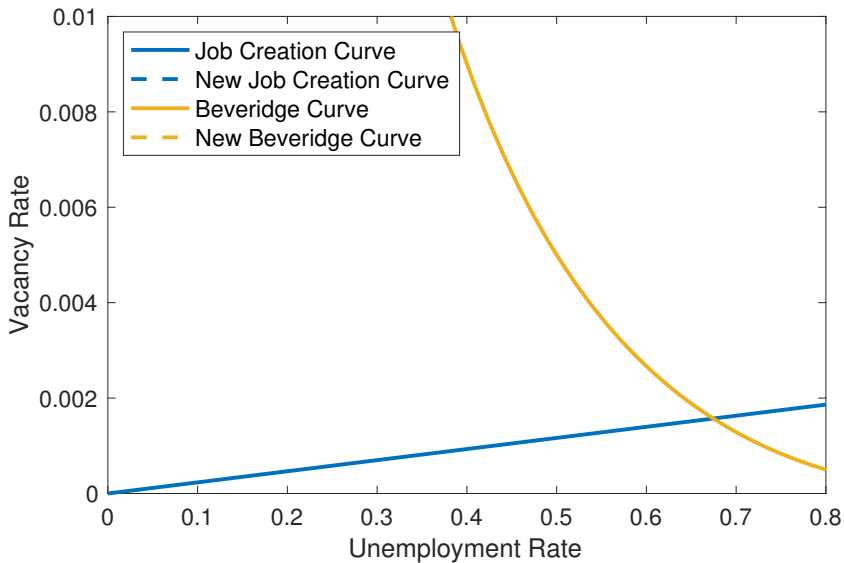
$$u = \frac{\delta}{\delta + p(\theta)}$$

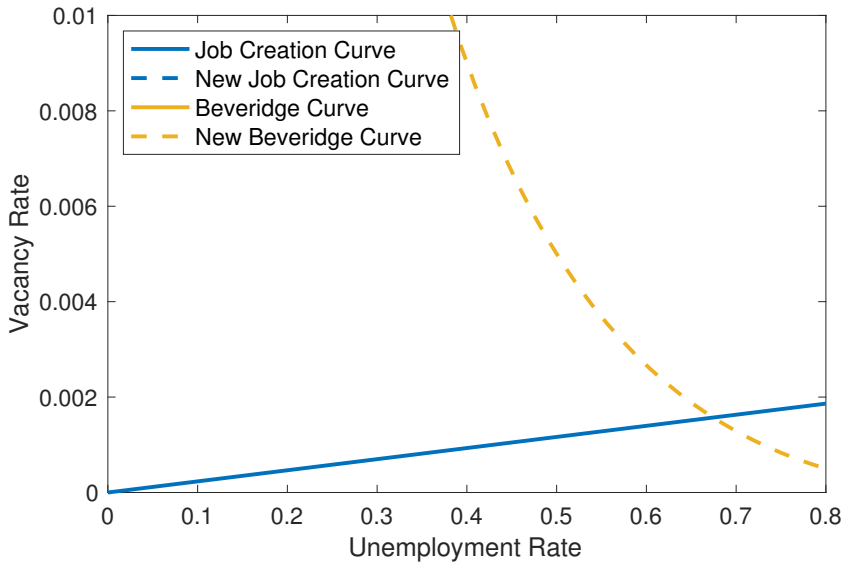
- Steady state? $w^* \downarrow$, $\theta^* \uparrow$, $v^* \uparrow$, $u^* \downarrow$

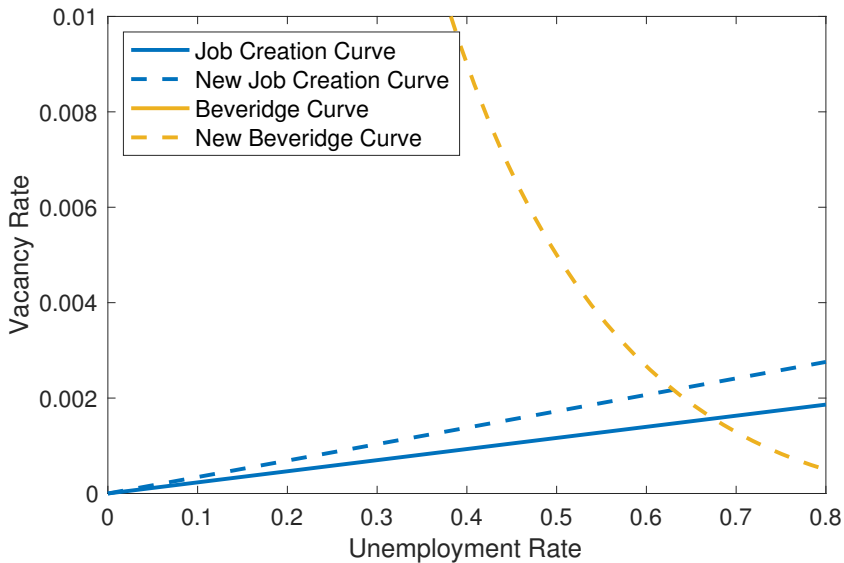












Efficiency

- Is zero unemployment efficient? **No**
 - higher unemployment incentivizes firms to post vacancies
 - but high unemployment is costly, less production
- Is a high vacancy rate efficient?
 - vacancy creation is costly
 - but lots of vacancies reduces unemployment
- So what is the efficient level of θ ?

Efficiency

- Congestion externality
 - one more hiring firm makes unemployed workers better off and makes all other hiring firms worse off
 - one more searching worker makes hiring firms better off and makes all other searching workers worse off
- Appropriability
 - firm pays a cost κ to post vacancy but does not get to keep the entire output y

Efficiency

- What value of θ would the social planner choose to maximize total output/utility if he is constrained by the same matching frictions?
 - does not care about wage b/c it's a linear transfer from the firm to the worker
- Does there exist a wage such that job creation is the same in the decentralized equilibrium as in the social planners outcome?
- Can we achieve this wage with the Nash solution?

Social Planner's Problem

$$\int_0^{\infty} e^{-rt} [y(1-u) + bu - \kappa\theta u] dt$$

s.t. $\dot{u} = \delta(1-u) - p(\theta)u$

- Social planner's problem
 - $y(1-u)$: social output of employment
 - bu : leisure enjoyed by unemployed workers
 - $\kappa\theta u$: cost of jobs
- Social planner is subject to the same transition equation for unemployment

Social Planner's Problem

- The Hamiltonian

$$H = e^{-rt}[y(1 - u) + bu - \kappa\theta u] + \mu(t)[\delta(1 - u) - \rho(\theta)u]$$

- FOCs

$$H_u = -\dot{\mu} \Rightarrow -e^{-rt}(y - b + \kappa\theta) - [\delta + \rho(\theta)]\mu + \dot{\mu} = 0$$

$$H_\theta = 0 \Rightarrow -e^{-rt}\kappa u - \mu u q(\theta)(1 - \beta) = 0$$

Social Planner's Problem

- Using $p(\theta) = \theta q(\theta)$ and solving in steady state ($\dot{\mu} = 0$)

$$(1 - \beta)(y - b) - \frac{\delta + r + \beta p(\theta)}{q(\theta)} \kappa = 0 \quad (1)$$

- From the decentralized solution, plug the wage curve into the Job creation curve

$$(1 - \gamma)(y - b) - \frac{\delta + r + \gamma p(\theta)}{q(\theta)} \kappa = 0 \quad (2)$$

Efficiency

- Comparing (1) and (2) we see that we have efficiency in the decentralized market if $\beta = \gamma$. The workers bargaining power is equal to the elasticity of the matching function with respect to u .
- Let $\eta(\theta)$ be the elasticity of the job filling rate ($q(\theta)$), the general result is that we have efficiency when

$$\eta(\theta) = \gamma$$

- This is called the Hosios (1990) condition