The Job Arrival Rate and DMP

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From last time

$$rU = b + \alpha \int_{w_R}^{\infty} E(w) - U \; dF(w)$$

$$rE(w) = w + \delta[U - E(w)]$$

- We now have some ways of thinking about $F(w)$
- What about α?
    - typical assumption: poisson arrival rate
    - what does it represent?
    - what does this arrival rate depend on?

The arrival rate of jobs

- At the beginning we assumed you get a job offer every period
  - with an exogenous wage distributions we have unemployment only because you received a job offer less than your reservation wage
- Now let's assume there is an arrival rate of jobs
  - frictional unemployment: unemployed because you did not get an offer
  - why did you not get an offer: coordination frictions

Coordination Frictions

- Trade in the labor market is decentralized
  - firms make decisions about how many jobs to create and wages to offer
  - workers make decisions about where to apply to job, how many jobs to apply to, ect ...
- More than 1 worker applies to the same job: unemployment
- No worker applies to a certain job: unfilled vacancy
- Burdett, Shi, Wright (2001)

Burdett, Shi, Wright

- Environment
  - Two workers (1 and 2) homogeneous and looking for work, each apply to only one job
  - Two firms (A and B) homogeneous and each have one job to fill
  - If the job is filled output $= y$, and wage $= w$
  - One shot game
- Payoffs
  - If a match occurs

$$u = w \quad \pi = y - w$$

  - If no match occurs

$$u = 0 \quad \pi = 0$$

Burdett, Shi, Wright

- Firms choose a wage to offer
  - w_A and w_B
- Workers choose which job to apply to
  - worker i applies to firm A with prob $= \theta_i$
  - worker i applies to firm B with prob $= 1 - \theta_i$
- Two stage game
  - Stage 1: Firms post wages
  - Stage 2: Workers choose probabilities

Stage 2

- Worker takes wages as given.
- Worker 1's utility from applying to firm A and firm B

$$U_{1A} = \frac{1}{2}\theta_2 w_A + (1 - \theta_2) w_A$$
$$U_{1B} = \theta_2 w_B + \frac{1}{2}(1 - \theta_2) w_B$$

- Worker 2's utilities are symmetric
- Worker 1 is indifferent between applying to both jobs ($U_{1A} = U_{1B}$) if

$$\theta(w_A, w_B) = \frac{2w_A - w_B}{w_A + w_B}$$

Stage 2

- Worker 1's strategy

$$\theta_1 \begin{cases} 0 & \text{if } \theta_2 > \theta(w_A, w_B) \\ 1 & \text{if } \theta_2 < \theta(w_A, w_B) \\ [0,1] & \text{if } \theta_2 = \theta(w_A, w_B) \end{cases}$$

- Worker 2's strategy is symmetric
- When does $\theta(w_A, w_B) = 1$?

$$w_A > 2w_B$$

- When does $\theta(w_A, w_B) = 0$?

$$w_A < \frac{1}{2}w_B$$

- When is $0 < \theta(w_A, w_B) < 1$?

$$\frac{1}{2}w_B < w_A < 2w_B$$

Stage 2

- If $w_A > 2w_B$, both workers are better off going to firm A

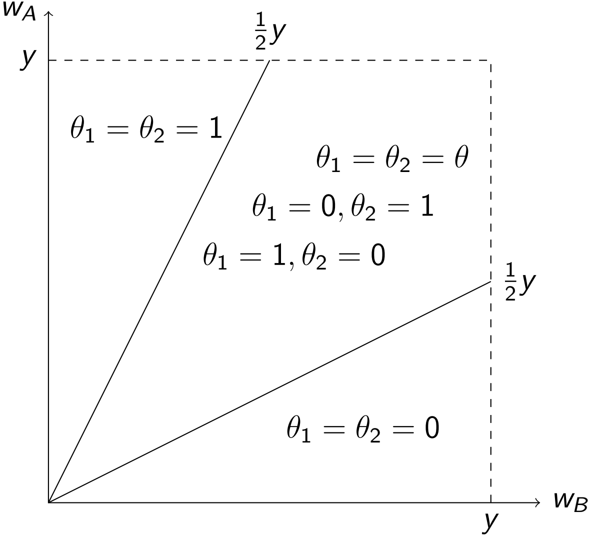
$$\theta_1 = \theta_2 = 1$$

- If $w_A < \frac{1}{2}w_B$, both workers are better off going to firm B

$$\theta_1 = \theta_2 = 0$$

- If $\frac{1}{2}w_B < w_A < 2w_B$ there are three equilibria
  - Pure strategy: (θ_1, θ_2) is $(0, 1)$ or $(1, 0)$
    - Perfect coordination
  - Mixed strategy: $\theta_1 = \theta_2 = \theta(w_A, w_B)$
    - Coordination frictions

Stage 2



w_A
$\frac{1}{2}y$
y
$\theta_1 = \theta_2 = 1$
$\theta_1 = \theta_2 = \theta$
$\theta_1 = 0, \theta_2 = 1$
$\theta_1 = 1, \theta_2 = 0$
$\frac{1}{2}y$
$\theta_1 = \theta_2 = 0$
w_B
y

Stage 1

- Taking the workers strategies as given solve for firm profits
  - If $w_A > \frac{1}{2}w_B$ then firm A gets both workers

$$\pi_A = y - w_A \ , \ \pi_B = 0$$

  - If $w_A < 2w_B$ then firm B gets both workers

$$\pi_A = 0 \ , \ \pi_B = y - w_B$$

  - If $\frac{1}{2}w_B < w_A < 2w_B$, in both pure strategy equilibria

$$\pi_A = y - w_A \ , \ \pi_B = y - w_B$$

Stage 1

- If it posts $\frac{1}{2}w_B < w_A < 2w_B$, and workers play a mixed strategy, Firm A's profits are:

$$\pi_A = (y - w_A)\theta_1(1-\theta_2) + (y - w_A)(1-\theta_1)\theta_2 + (y - w_A)\theta_1\theta_2 + 0(1-\theta_1)(1-\theta_2)$$

$$\pi_A = (y - w_A)\frac{3w_B(2w_A - w_B)}{(w_A + w_B)^2}$$

- Firm A's profit maxing best response:

$$w_A^*(w_B) = \frac{w_B(4y + w_B)}{5w_B + 2y}$$

- Firm B's profits and best response are symmetric

Mixed Strategy Equilibrium

- Solving for the equilibrium

$$\theta_1 = \frac{1}{2} \qquad \theta_2 = \frac{1}{2}$$

$$w_A = \frac{y}{2} \qquad w_B = \frac{y}{2}$$

$$\pi_A = \frac{3y}{8} \qquad \pi_B = \frac{3y}{8}$$

Mixed Strategy Equilibrium

- The expected number of matches

$$M = 1\theta_1\theta_2 + 1(1-\theta_1)(1-\theta_2) + 2(1-\theta_1)(\theta_2) + 2\theta_1(1-\theta_2)$$

- Expected probability of receiving job offer (assuming that the firm randomly chooses between the two workers if both apply to the same job)

$$\alpha = \frac{M}{2} = 0.75$$

General Solution

- Burdett, Shi, Wright show that for m firms and n workers the matching functions is:

$$M(m,n) = m\left[1 - \left(1 - \frac{1}{m}\right)^n\right]$$

- Arrival rate of job offers $M(m,n)/n$
- Fix $n/m = b$ then as m increases the arrival rate decreases $\Rightarrow$ matching function is decreasing returns to scale. Bigger markets have larger frictions
- As $m \to \infty$ matching function converges to constant returns to scale

The Matching Function

- Typically in random search models we do not explicitly model the application strategies of workers.
- Reduced form approach to matching friction: assume a matching function exists
- Matching function:
  - depends on the number of unemployed U and vacancies V
  - depends on some aggregate efficiency parameter A
  - exhibits constant returns to scale
$$M(U,V) = AU^{\beta}V^{1-\beta}$$
- Nice discussion: Petrongolo & Pissarides (2001)

The Job Finding and Filling Rate

- Given the matching function

$$M(U, V) = AU^{\beta}V^{1-\beta}$$

define labor market tightness $\theta = V/U$

- The job finding rate

$$p(\theta) = \frac{M(U, V)}{U} = A\theta^{1-\beta}$$

- The job filling rate

$$q(\theta) = \frac{M(U, V)}{V} = A\theta^{-\beta}$$

Diamond Mortensen Pissarides (DMP)

- Environment
  - continuous time
  - everyone discounts at rate r
  - homogeneous workers searching for jobs
  - homogeneous firms post vacancies
  - job finding and filling rates determined by matching function
  - wages determined by Nash Bargaining

Diamond Mortensen Pissarides (DMP)

- A steady state
  - a measure of unemployed workers u
  - a measure of vacancies v
  - a wage w
- We have three unknowns so we will have three steady state equations to solve
  (1) The Beveridge Curve: a relationship between the unemployment rate and vacancy rate
  (2) Job Creation: firms continue to post vacancies until the value of having a vacant job is zero
  (3) The Nash solution: gives a solution to the wage as a function of labor market tightness
- (2) and (3) will determine the steady state values of θ^* and w^*. Given θ^*, (1) and (3) will determine the steady state values of u^* and v^*

Workers

- When unemployed, workers receive unemployment benefits b and search for jobs
- The value of unemployment is

$$rU = b + p(\theta)[E - U]$$

- When employed workers receive wage w and lose their jobs at an exogenous rate δ
- The value of employment is

$$rE = w + \delta[U - E]$$

Workers

- Solving the value of unemployment and value of employment simultaneously gives:

$$E = \frac{w(r + p(\theta)) + \delta b}{r(r + p(\theta) + \delta)}$$

$$U = \frac{(r + \delta)b + p(\theta)w}{r(r + p(\theta) + \delta)}$$

Firms

- If a firm has a vacant job it pays flow cost κ to post the vacancy

- The value of having a vacancy is

$$rV = -\kappa + q(\theta)[J - V]$$

- If a firm has a filled job it produces output y and pays wage w, the job is exogenously destroyed at rate δ

- The value of a filled job is

$$rJ = y - w + \delta[V - J]$$

Beveridge Curve

- In steady state the inflow and outflow of unemployment are equal
  - Inflow: $\delta(1-u)$
  - Outflow: $p(\theta)u$
- Solving for u gives the unemployment rate

$$u = \frac{\delta}{\delta + p(\theta)}$$

- Since $\theta = v/u$, where v is the vacancy rate, this gives us a relationship between u and v known as the Beveridge Curve.

Job Creation Curve

- The free entry condition means that firms will continue to post vacancies until the value of a vacancy is driven to zero. This implies:

$$V = 0$$

$$J = \frac{y - w}{r + \delta} \quad \& \quad J = \frac{\kappa}{q(\theta)}$$

- Equating the two values for a filled job gives us the second equation we need

$$y - w - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

Wages

- Wages are determined by bargaining between the firm and the worker to avoid the critiques raised by Rothchild and Diamond
- The bargaining problem
  - Total value of a match

$$\Omega = E(w) + J(w)$$

  - Disagreement values: (U, V)
  - Bargaining power: γ
- Generalized Nash Bargaining problem

$$w = \underset{w}{\operatorname{argmax}} [E(w) - U]^{\gamma} [J(w)]^{1-\gamma}$$

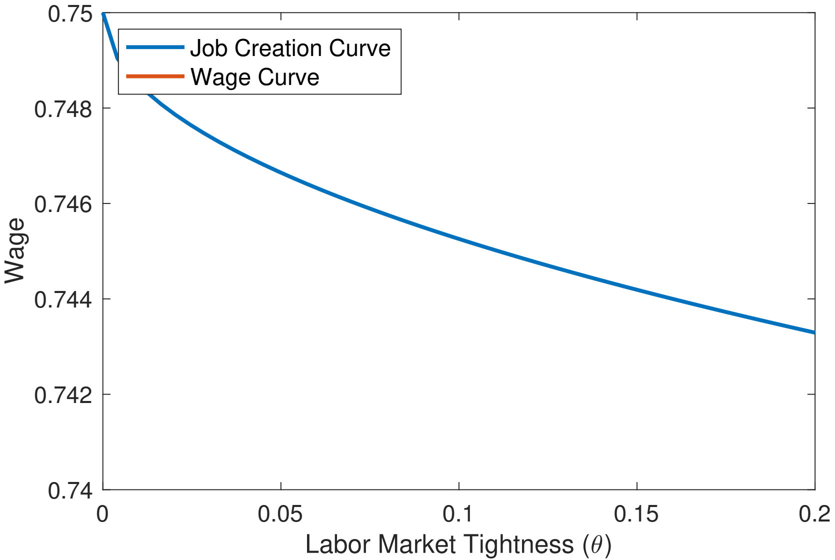
Wages

- Plugging in the value functions and solving for the max gives us the last equation we need to find the steady state

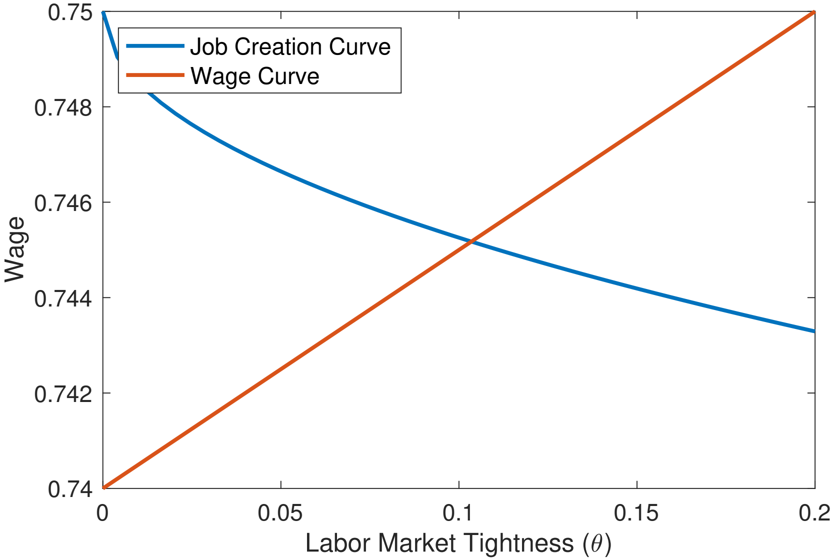
$$w = \gamma y + (1-\gamma) rU$$

- Use FOC, the value function for rU, and $J = \kappa / q(\theta)$ to get:

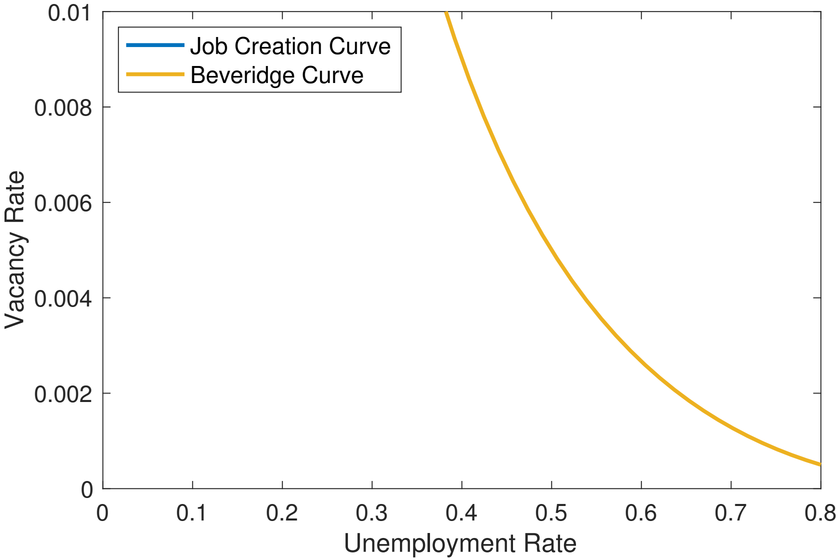
$$w = (1-\gamma) b + \gamma (y + \kappa \theta)$$



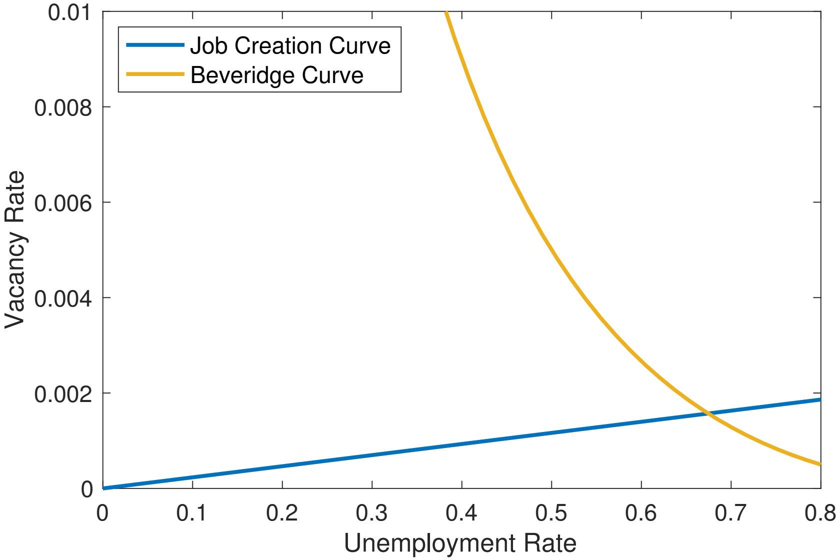
$$V = 0 \Rightarrow J = \frac{y - w}{r + \delta} \quad \& \quad J = \frac{\kappa}{q(\theta)}$$



$$w = (1 - \gamma)b + \gamma(y + \kappa\theta)$$



$$u = \frac{\delta}{\delta + p(v/u)}$$



Job Creation Curve
Beveridge Curve
Vacancy Rate
Unemployment Rate
0.01
0.008
0.006
0.004
0.002
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

$$v/u = \theta^*$$

Comparative Statics

- What will happen if we decrease the worker's bargaining power (γ)?
  - Job Creation?

$$y - w - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

  - Wages?

$$w = (1 - \gamma)b + \gamma(y + \kappa\theta)$$

  - Beveridge Curve?

$$u = \frac{\delta}{\delta + p(\theta)}$$

  - Steady state?

Comparative Statics

- What will happen if we decrease the worker's bargaining power (γ)?
  - Job Creation?

$$y - w - \frac{\kappa(r+\delta)}{q(\theta)} = 0$$

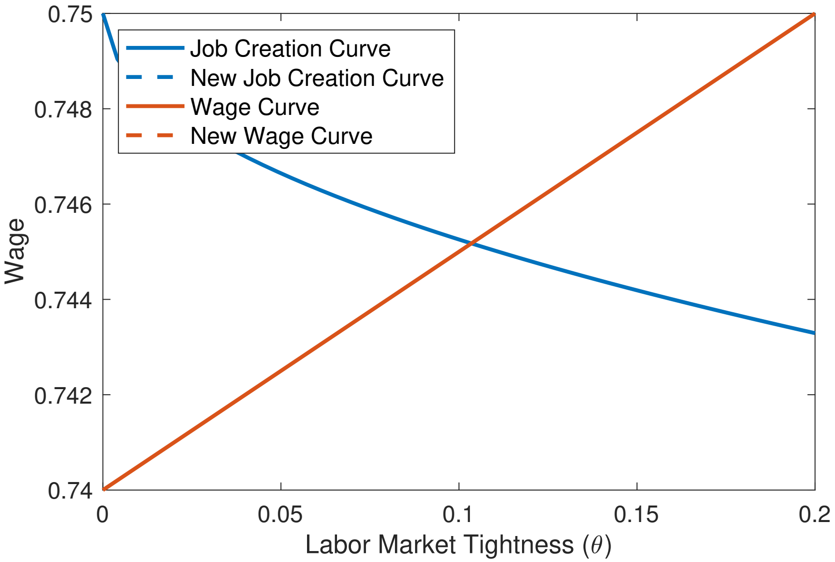
  - Wages?

$$w = (1-\gamma)b + \gamma(y + \kappa\theta)$$

  - Beveridge Curve?

$$u = \frac{\delta}{\delta + p(\theta)}$$

  - Steady state? $w^* \downarrow$, $\theta^* \uparrow$, $v^* \uparrow$, $u^* \downarrow$


Job Creation Curve
New Job Creation Curve
Wage Curve
New Wage Curve
Wage
Labor Market Tightness (θ)
0.75
0.748
0.746
0.744
0.742
0.74
0
0.05
0.1
0.15
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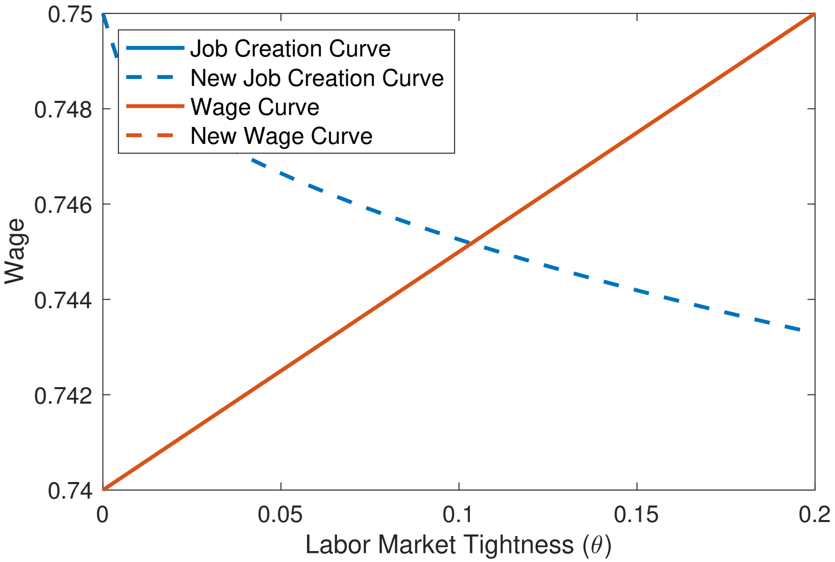

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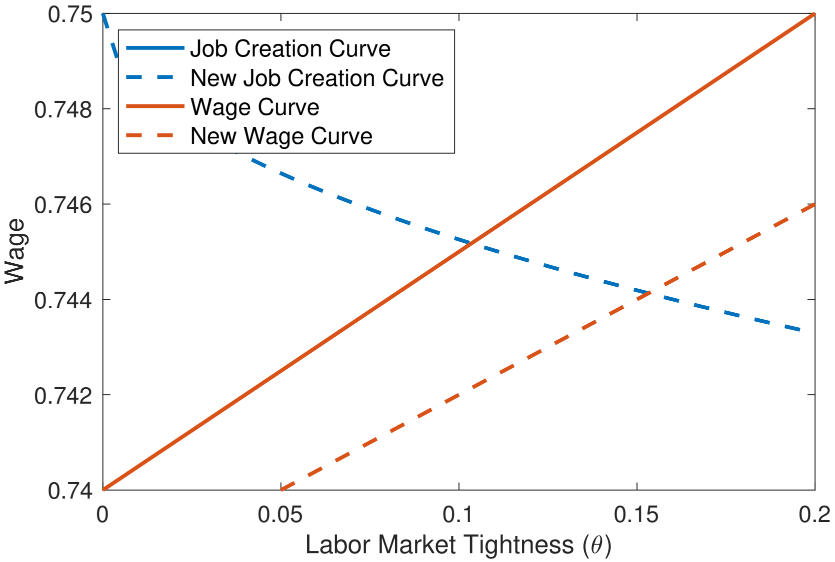
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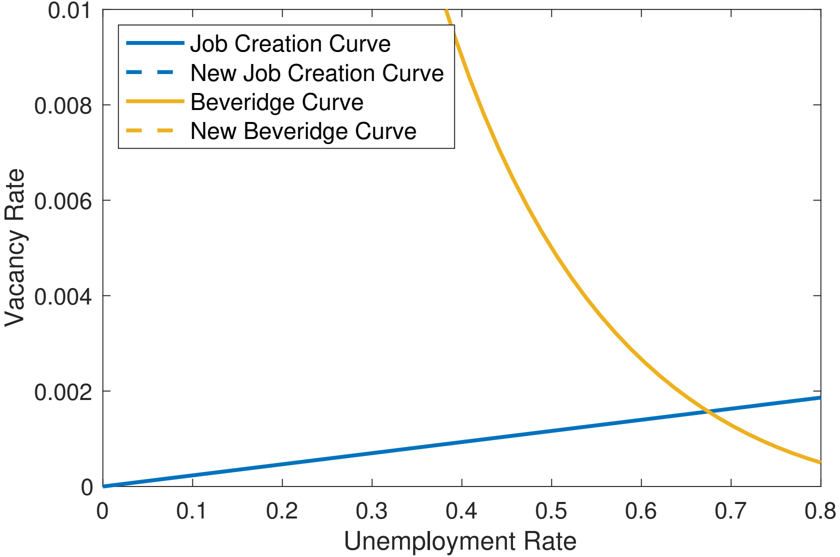

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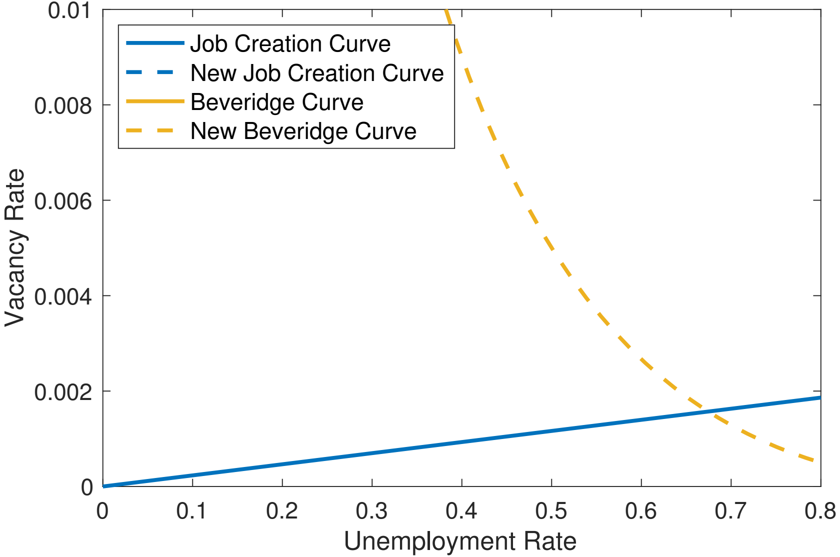
Unemployment Rate



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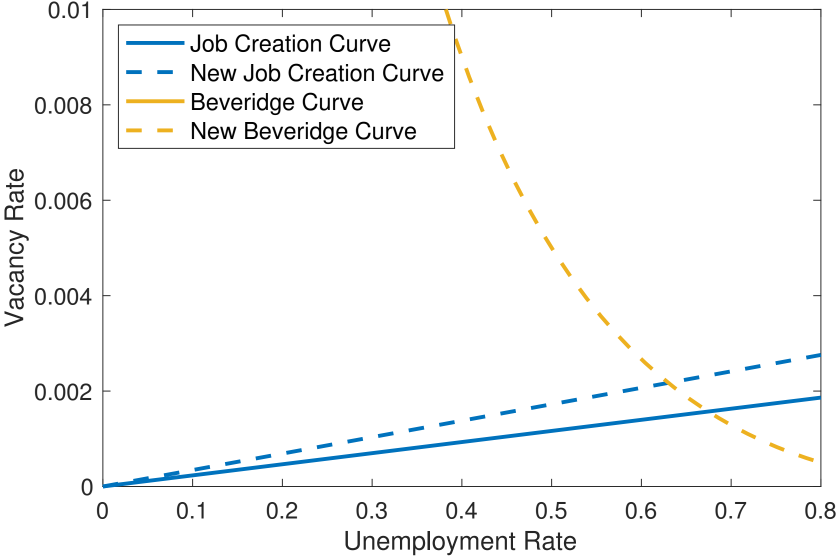

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Efficiency

- Is zero unemployment efficient? No
  - higher unemployment incentivizes firms to post vacancies
  - but high unemployment is costly, less production
- Is a high vacancy rate efficient?
  - vacancy creation is costly
  - but lots of vacancies reduces unemployment
- So what is the efficient level of θ?

Efficiency

- Congestion externality
  - one more hiring firm makes unemployed workers better off and makes all other hiring firms worse off
  - one more searching worker makes hiring firms better off and makes all other searching workers worse off
- Appropriability
  - firm pays a cost κ to post vacancy but does not get to keep the entire output y

Efficiency

- What value of θ would the social planer choose to maximize total output/utility if he is constrained by the same matching frictions?
  - does not care about wage b/c it's a linear transfer from the firm to the worker
- Does there exist a wage such that job creation is the same in the decentralized equilibrium as in the social planners outcome?
- Can we achieve this wage with the Nash solution?

Social Planner's Problem

$$\max_{u,\theta} \int_0^\infty e^{-rt}[y(1-u) + bu - \kappa\theta u] \; dt$$

$$\text{s.t.} \quad \dot{u} = p(\theta)u - \delta(1-u)$$

- Social planner's problem
  - $y(1-u)$: social output of employment
  - bu: leisure enjoyed by unemployed workers
  - $\kappa\theta u$: cost of jobs
- Social planner is subject to the same transition equation for unemployment

Social Planner's Problem

- The Hamiltonian

$$H = e^{-rt}[y(1-u)+bu-\kappa\theta u]+\mu(t)[\delta(1-u)-p(\theta)u]+\dot{\mu}(t)u$$

- FOCs

$$H_u = 0 \Rightarrow \qquad -e^{-rt}(y-b+\kappa\theta)-[\delta+p(\theta)]\mu+\dot{\mu}=0$$
$$H_\theta = 0 \Rightarrow \qquad -e^{-rt}\kappa u-\mu u q(\theta)(1-\beta)=0$$

- Solve for costate variable

$$\mu = \frac{-e^{-rt}\kappa}{q(\theta)(1-\beta)}$$

- Solve for $\dot{\mu} = d\mu/dt$

$$\dot{\mu} = -r\mu$$

Social Planner's Problem

- Using $p(\theta) = \theta q(\theta)$ and plugging into $H_u = 0$

$$(1-\beta)(y-b) - \frac{\delta + r + \beta p(\theta)}{q(\theta)}\kappa = 0 \tag{1}$$

- From the decentralized solution, plug the wage curve into the Job creation curve

$$(1-\gamma)(y-b) - \frac{\delta + r + \gamma p(\theta)}{q(\theta)}\kappa = 0 \tag{2}$$

Efficiency

- Comparing (1) and (2) we see that we have efficiency in the decentralized market if $\beta = \gamma$. The workers bargaining power is equal to the elasticity of the matching function with respect to *u*.

- Let $\eta(\theta)$ be the elasticity of the job filling rate ($q(\theta)$), the general result is that we have efficiency when

$$\eta(\theta) = \gamma$$

- This is called the Hosios (1990) condition