Nash Bargaining

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How do we get a wage distribution?

- **Problem:** Rothschild critique & Diamond Paradox
  - even with job heterogeneity
- Firms choose wages to max profits
  - Burdett-Judd (1983): multiple applications
  - Albrecht-Axell (1984): heterogeneity in b
  - Burdett-Mortensen (1998): on-the-job search
- Firm and worker bargain over wage
  - Rubinstein's alternating offers
  - Nash Bargaining

Bargaining Theory

- Strategic Bargaining:
  - explicitly model the bargaining process in game form
  - consider the equilibrium of the game

  eg: Rubinstein's Alternating Offers (1982)
- Axiomatic Bargaining:
  - abstract from specifics about the bargaining process
  - consider solutions that satisfy reasonable properties

  eg: Nash Bargaining (1950)

Alternating Offers

- Environment:
  - Two players bargain over a "pie" of size 1
  - Each player only cares about his share
  - Set of all possible solutions:

$$X = \{(x_1, x_2) : x_1 + x_2 = 1 \text{ and } x_i \geq 0, i = 1, 2\}$$

  - x_i is player i's share of the pie
  - Time is infinite, $t \in T = \{1, 2, 3, \dots\}$
  - Bargaining breaks down with prob. α after each t
  - If bargaining breaks down the outcome is $(0, 0)$

Alternating Offers

- Bargaining Procedure:
  - At $t = 1$ player 1 proposes a split $\hat{x} = (\hat{x}_1, \hat{x}_2)$
  - At $t = 1$ player 2 accepts or rejects offer
  - If reject: with probability $1 - \alpha$ bargaining continues
  - At $t = 2$ player 2 proposes a split $\tilde{x} = (\tilde{x}_1, \tilde{x}_2)$
  - At $t = 2$ player 1 accepts or rejects offer
  - If reject: with probability $1 - \alpha$ bargaining continues
  - ......

Alternating Offers

- A simple set of strategies
  - Player 2 accepts $\hat{x}$ if:

$$\hat{x}_2 \geq (1-\alpha)\tilde{x}_2$$

  - Player 1 accepts $\tilde{x}$ if:

$$\tilde{x}_1 \geq (1-\alpha)\hat{x}_1$$

- Rubinstein (1982): *These strategies constitute the unique subgame perfect equilibrium of the infinitely repeated alternating offers game with breakdown.*

Alternating Offers

- Solution:

$$\hat{x} = \left(\frac{1}{2-\alpha}, \frac{1-\alpha}{2-\alpha} \right)$$

$$\tilde{x} = \left(\frac{1-\alpha}{2-\alpha}, \frac{1}{2-\alpha} \right)$$

- $\hat{x}$ is the solution if player 1 makes first offer
  - first mover advantage:

$$\frac{1}{2-\alpha} > \frac{1-\alpha}{2-\alpha}$$

Alternating Offers

How does this map into a job search model?

- Value of employment at wage w

$$rE(w) = w + \delta[U - E(w)]$$

$$E(w) = \frac{w + \delta U}{r + \delta} \tag{1}$$

- Value of a filled job at wage w

$$rJ(w) = p - w - \delta J(w)$$

$$J(w) = \frac{p - w}{r + \delta} \tag{2}$$

Alternating Offers

How does this map into a job search model?

- Firm and worker bargain over the wage
  - w^f: firm's wage offer
  - w^w: worker's wage offer
- Strategies:
  - Worker accepts firm's offer if:

$$E(w^f) \geq \alpha U + (1-\alpha)E(w^w)$$

  - Firms accepts worker's offer if:

$$J(w^w) \geq (1-\alpha)J(w^f)$$

Alternating Offers

How does this map into a job search model?

- Using (1) and (2) the subgame perfect equilibrium is:

$$w^f = \frac{1-\alpha}{2-\alpha}p + \frac{1}{2-\alpha}rU$$

$$w^w = \frac{1}{2-\alpha}p + \frac{1-\alpha}{2-\alpha}rU$$

- Assuming $p > rU$ and $\alpha > 0$ first mover has advantage

- $w^f = w^w$ if $\alpha = 0$ (symmetric Nash Bargaining Solution)

Axiomatic Bargaining

- Same situation as before
  - Two players bargaining over a “pie” of size 1
- Consider these 4 axioms:
  1. Pareto Efficiency: no one can be made better of without make someone else worse off
  2. Symmetry: If players are the same, the solution should not discriminate between them
  3. Invariant to Affine Transformation: affine transformation of payoffs and disagreement values does not change the solution
  4. Independence of Irrelevant Alternatives: If the solution x^* from a set A and is an element of subset $B \subset A$, then x^* must be chosen from B.

Axiomatic Bargaining

- The bargaining model
  - Two players: $1, 2$
  - A set of feasible agreements:

$$X = \{(x_1, x_2) \in \text{bounded and convex set}\}$$

$$X = \{(x_1, x_2) : x_1 + x_2 = 1 \text{ and } x_i \geq 0, i = 1, 2\}$$

  - The disagreement outcome $(d_1, d_2) = (0, 0)$

Nash Bargaining Solution

- Nash Bargaining Solution (NBS) is the unique solution that satisfies the 4 axioms

  Definition: The payoff $x^* = (x_1^*, x_2^*)$ is a Nash Bargaining Solution if it solves:

$$\max_{x_1, x_2} (x_1 - d_1)(x_2 - d_2)$$

$$\text{s.t. } (x_1, x_2) \in X$$
$$(x_1, x_2) \geq (d_1, d_2)$$

Nash Bargaining Solution

- The first order condition solves the Nash Bargaining Solution

$$\max_{x_1}(x_1 - 0)(1 - x_1 - 0)$$

$$[\text{FOC}] : 1 - 2x_1 = 0$$

$$x_1^* = \frac{1}{2}\ ,\ x_2^* = \frac{1}{2}$$

Nash Bargaining Solution

How does this map into a job search model?

- The disagreement point:

$$(d_w, d_f) = (U, 0)$$

- The bargaining set:

$$X = \left\{ \big(E(w), J(w)\big) : E(w) + J(w) - U = \Omega \,,\; E(w) \geq U \,,\; J(w) \geq 0 \right\}$$

- The optimization:

$$\max_w \big(E(w) - U\big)\big(J(w)\big)$$

$$\max_w \left(\frac{w - rU}{r + \delta}\right)\left(\frac{p - w}{r + \delta}\right)$$

(take the log to solve!)

Nash Bargaining Solution

How does this map into a job search model?

- The symmetric Nash Bargaining Solution

$$w^* = \frac{1}{2}p + \frac{1}{2}rU$$

- Does Axiom 2 (Symmetry) make sense here?
  - Are the worker and firm identical?
  - Does one have more bargaining power?

Nash Bargaining Solution

The Generalized Solution

- Let γ be the worker's bargaining power
- Disagreement point and bargaining set same as before
- The optimization

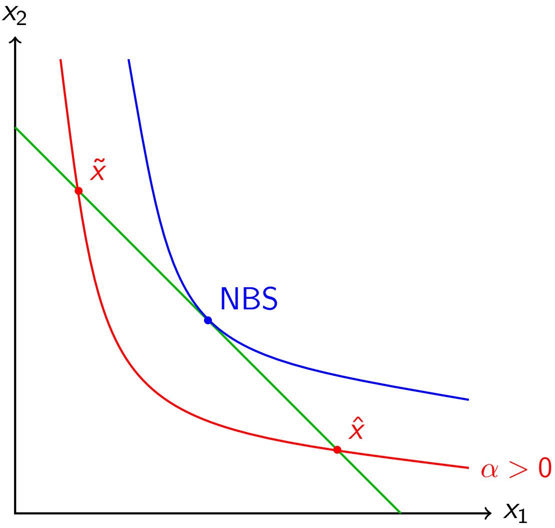
$$\max_{w} \left(E(w) - U\right)^{\gamma} \left(J(w)\right)^{1-\gamma}$$

- The Generalized Nash Bargaining Solution

$$w^{*} = \gamma p + (1-\gamma) rU$$

- What happens as $\gamma \to 1$? $\gamma \to 0$?

Convergence of Alternating Offers to NBS



Convergence of Alternating Offers to GNBS

- Alternating offers game with discounting
- Discount rates $\delta_1 \neq \delta_2$
  - different degrees of patience
  - different risk aversion
- $\delta_i = e^{-p_i \Delta}$
- As $\Delta \to 0$ solution converges to GNBS

For the Assigment

- Jobs are heterogeneous in productivity:

$$\theta \sim G(\theta)$$

- On matching the productivity of a job is realized and bargaining begins

- Wage distribution is a transformation of productivity distribution