The Wage Distribution

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Homework Answers

1. Write down the value function for employment and unemployment if you have a probability δ of losing your job every period.

$$U = b + \beta \int \max\{U, E(w)\}\ dF(w)$$

$$E(w) = w + \beta[\delta U + (1 - \delta)E(w)]$$

Homework Answers

2. Derive the continuous time value functions if δ is the poisson rate of losing your job

$$E(w) = \frac{wdt + \delta dtU + (1-\delta dt)E(w)}{1+rdt}$$
$$rdtE(w) = wdt + \delta dt[U - E(w)]$$
$$rE(w) = w + \delta[U - E(w)]$$

$$U = \frac{bdt + \alpha dt \int \max\{U, E(w)\}\ dF(w) + (1-\alpha dt)U}{1+rdt}$$
$$rdtU = bdt + \alpha dt \int \max\{U, E(w)\}\ dF(w) - \alpha dtU$$
$$rdtU = bdt + \alpha dt \int \max\{0, E(w) - U\}\ dF(w)$$
$$rU = b + \alpha \int_{w_R} [E(w) - U]\ dF(w)$$

Where does $F(w)$ come from?

- are firms posting wages to maximizes profits?
  - why would firms post different wages? heterogeneity?
  - Rothschild critique
  - Diamond paradox

Example

- Workers
  - unit mass of identical workers
  - flow value of unemployment $b = 0$
  - workers search for jobs
  - once a worker accepts a new worker is born and searches

Example

- Firms
  - a continuum of firms with different productivities
  - $y \in [0, \infty)$ is productivity drawn from c.d.f. $G(y)$
  - firms post single vacancy at cost $\gamma > 0$
  - filled jobs last forever
  - discount at rate β
  - price of output normalized to 1

Example....the issue

- what does the wage offer distribution look like?
- if firms post wages to max profits: it will be degenerate!

Example

- Worker's Problem
  - choose whether or not to accept an offer

$$a\colon \mathbb{R}_+ \to [0,1]$$

  - from before:

$$a(w) = \begin{cases} 1 & \text{if } w \geq w_R \\ 0 & \text{otherwise} \end{cases}$$

Example

- Firm's Problem
  - given the workers strategy $a(w)$ the firm chooses
    - to post a vacancy

$$p \colon Y \to \{0, 1\}$$

    - the wage to post

$$w \colon Y \to \mathbb{R}_+$$

  - given the decision to post, firms max profits

$$\max_{w} \quad \pi(y)$$

$$\max_{w} \quad \frac{1}{n} \frac{(y - w)}{1 - \beta} - \gamma$$

$$\text{s.t. } w \geq w_R \quad \& \quad n = \int p(y)\, dG(y)$$

Example

- Firms solution
  - the wage decision

$$w(y) = w_R$$

  - the posting decision

$$p(y) = \begin{cases} 1 & \text{if } \pi(y) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- The wage distribution
  - Rothschild critique: it's degenerate! $F(w(y)) = w_R \ \forall y$
  - Diamond paradox: all firms offer $b = w_R$

How do we get a wage distribution?

- Firms choose wages to max profits
  - Albrecht-Axell (1984): heterogeneity in b
  - Burdett-Judd (1983): multiple applications
  - Burdett-Mortensen (1998): on the job search

The Search Environment

- Assumptions about the search process
  - **Sequential Search:** Workers receive offers sequentially (typically the cost of search is time rather than a monetary cost). ex: McCall model
  - **Non-sequential Search:** Workers choose the number of applications to send at a cost c per application, then choose the highest wage offer. ex: Stigler
- Burdett-Judd (1983): non-sequential search

Burdett-Judd

- **The setup:** One-shot game with a continua of workers and firms
  - Workers: decide how many wage offers to sample
  - Firms: decide what wage to offer
- **Environment**
  - μ: measure of job seekers relative to firms
  - p: revenue per employee
  - b: workers value of leisure
  - c: cost per additional application (first application is free)

Equilibrium

- **Equilibrium Objects:**
  - $\{q_N\}_{n=1}^{\infty}$: fraction of workers sampling n wages
  - w_R: reservation wage
  - $F(w)$: distribution of wage offers
  - $\pi(w)$:expected profit at w
- **Definition:** An equilibrium is the set of objects above s.t.,
  1. Given $\{q_N\}_{n=1}^{\infty}$ and w_R

$$\pi(w) = \pi \ \forall \ w \text{ in the support of } F$$
$$\pi(w) < \pi \ \forall \ w \text{ not in the support of } F$$

  2. Given $F(w)$, w_R is optimal and $\{q_N\}_{n=1}^{\infty}$ is generated by the income-maximizing strategies of workers.

Firms Strategies

- Take workers strategies $\{q_N\}_{n=1}^{\infty}$ as given. What possible wages will the firm post?

1. $q_1 = 1$: all workers only sample one wage

2. $q_1 = 0$: all workers sample more than one wage

3. $q_1 \in (0, 1)$: some workers sample one wage

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  3. $q_1 \in (0, 1)$: some workers sample one wage

     $\Rightarrow$ $F(w)$ is continuous with compact support $[b, \bar{w}]$ where $\bar{w} < p$

Understanding $F(w)$

- $F(w)$ is continuous: suppose there is an atom in $F(w)$ at $\tilde{w}$. Then a firm could increase profits by offering $\tilde{w} + \varepsilon$.
- $\bar{w} < p$: If some workers only sample one wage, $q_1 > 0$ then $w = p$ can not be optimal.
- b is the lower bound of the support of $F(w)$: Suppose $\underline{w} > b$, any worker willing to accept $\underline{w}$ in equilibrium would also be willing to accept $\underline{w} - \varepsilon$.

What do firm profits look like?

- If the firm choses $w = b$, only get workers who sample one wage

$$\pi(b) = \mu q_1(p - b)$$

- If the firm chooses $w = \bar{w}$, can attract all workers

$$\pi(\bar{w}) = \mu(p - \bar{w}) \sum_{n=1}^{\infty} n q_n$$

- But in equilibrium all firms must make the same profit

$$\pi(b) = \pi(\bar{w}) = \pi(w) \;\; \forall \; w \text{ in the support of F(w)}$$

Workers Strategy

- Suppose all firms offer $w = b$:

- Suppose all firms offer $w = p$:

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- Suppose all firms offer $w = p$:

  $\Rightarrow$ $q_1 = 1$, all workers sample one wage. But for all firms to offer $w = p$ it must be that no worker samples one wage ($q_1 = 0$). There is never a competitive equilibrium.

Workers Strategy

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- Suppose there exist a wage distribution $F(w)$

  $\Rightarrow$ Since workers are identical they all sample the same number of wage (not an equailibrium!) or they are indifferent between sampling n or $n+1$ number of wage.

  $\Rightarrow$ Since $q_1 \in (0,1)$ for there to be a wage distribution it must be that $q_1 + q_2 = 1$

Characterizing $F(w)$

- Fix $q_1 \in (0,1)$

$$\pi(w) = (p - w)\mu(q_1 + 2(1 - q_1)F(w))$$

- Since profits are equal for all $w \in [b, \bar{w}]$

$$\pi(b) = \pi(w) \Rightarrow (p - b)\mu q_1 = (p - w)\mu(q_1 + 2(1 - q_1)F(w))$$

$$F(w) = \frac{q_1(w - b)}{2(p - w)(1 - q_1)}$$

- Still missing q_1

Solving for q_1

- Marginal benefit of sampling 2 wages instead of 1 must equal c

$$V(q_1) = 2\int_b^{\bar{w}} wf(w)F(w)\ dw - \int_b^{\bar{w}} wf(w)\ dw$$

where $f(w)$ and $F(w)$ are functions of q_1

- Two solutions for $V(q_1) = c$, $V(q_1) \to 0$ as $q_1 \to 1$ or 0
    - Suppose q_1 is close to zero, then almost all wages close to p, little benefit to sending a second application
    - Suppose q_1 is close to one, then almost all wage close to b, little benefit to sending second application

Burdett-Mortensen (1998)

- **Key Idea:** On the job search generates a continuous wage distribution with no mass points.
- **Intuition:** High wage firms earn less profit per worker but attract more workers so equilibrium profits for firms are equal across the wage distribution.
- **Limits of the model:** Diamond outcome is the limit as on-the-job search disappears and competitive equilibrium as search frictions disappear.

Environment

- set in continuous time
- measure m of workers
- workers and firms are identical and discount the future at rate r
- workers are either employed or unemployed and receive job offers at poisson rate
  - λ_0 when unemployed
  - λ_1 when employed
- workers draw wage offers from known distribution $F(w)$
- workers receive b when unemployed
- workers lose their jobs at rate δ

Workers

- Unemployed

$$rU = b + \lambda_0 \Bigg[\qquad\qquad\qquad\qquad\qquad\qquad \Bigg]$$

- Employed

$$rE(w) = w + \lambda_1 \Bigg[\qquad\qquad\qquad\qquad\qquad\qquad \Bigg] + \delta[\qquad\qquad]$$

Workers

- Unemployed

$$rU = b + \lambda_0 \left[\int \max\{U, E(w)\} \; dF(w) - U \right]$$

$$rU = b + \lambda_0 \int_{R}^{\bar{w}} E(w) - U \; dF(w)$$

- Employed

$$rE(w) = w + \lambda_1 \left[\int \max\{E(w), E(w')\} \; dF(w') - E(w) \right] + \delta[U - E(w)]$$

$$rE(w) = w + \lambda_1 \int_{w}^{\bar{w}} E(w') - E(w) \; dF(w') + \delta[U - E(w)]$$

Firms

- Firms choose w to maximize their profits

$$\pi = \max_{w}(p - w)\ell(w|R, F)$$

- w determines
  - the revenue per worker $(p - w)$
  - the number of workers $\ell(w|R, F)$

Steady State and Equilibrium

- **Steady State:**
  - an unemployment rate that does not change
  - a distribution of wages paid $G(w)$
- **Equilibrium Objects:**
  - offered wage distribution $F(w)$
  - the reservation wage R
  - the profits of firms π
- **Equilibrium Definition:** the set of objects s.t. R is the reservation wage of the workers and profits are equal for all wages in the support of $F(w)$.

Steady State - Unemployment Rate

- in steady state the number of unemployed does not change
  - inflow: $\delta(m - u)$
  - outflow: $\lambda_0[1 - F(R)]u$
- the steady state number of unemployed

$$u = \frac{\delta m}{\delta + \lambda_0[1 - F(R)]}$$

- the steady state unemployment rate is u/m

Steady State - Distribution of Wages Paid

- The measure of workers earning wage $\leq w$ at time t is

$$G(w,t)[m-u(t)]$$

- In steady state $G(w,t)$ does not change

$$\begin{aligned} 0 &= \frac{\partial G(w,t)}{\partial t} \\ &= \lambda_0[F(w)-F(R)]u - [\delta + \lambda_1(1-F(w))]G(w)(m-u) \end{aligned}$$

- solving for $G(w)$ gives

$$G(w) = \frac{\delta[F(w)-F(R)]/[1-F(R)]}{\delta + \lambda_1[1-F(w)]}$$

Labor Supply

- To solve for the equilibrium wage distribution $F(w)$ we need to maximize profits of firms. For this we need labor supplied to each firm. Consider a firm paying w:

$$\ell(w|R,F) = \lim_{\varepsilon \to 0} \frac{G(w) - G(w-\varepsilon)}{F(w) - F(w-\varepsilon)}(m-u)$$

  - $[G(w) - G(w-\varepsilon)](m-u)$: steady state number of workers earning wage $\in [w, w+\varepsilon]$
  - $F(w) - F(w-\varepsilon)$: measure of firms offering wage $\in [w, w+\varepsilon]$

Labor Supply

- The labor supplied to a firm offering $w \geq R$ is

$$\ell(w|R,F) = \frac{\delta m \lambda_0[\delta + \lambda_1(1 - F(R))]/[\delta + \lambda_0(1 - F(R))]}{[\delta + \lambda_1(1 - F(w))]^2}$$

- The labor supplies to a firm offering $w < R$ is

$$\ell(w|R,F) = 0$$

- $\ell(w|R,F)$ is increasing in w and continuous unless $F(w)$ has a mass point

The Reservation Wage

- The reservation wage R is such that $E(R) = U$, so

$$R - b = (\lambda_0 - \lambda_1) \int_R^{\bar{w}} [E(w) - U] \, dF(w)$$

- Then integration by parts

$$\begin{aligned} R - b &= (\lambda_0 - \lambda_1) \int_R^{\bar{w}} E'(w)[1 - F(w)] \, dw \\ &= (\lambda_0 - \lambda_1) \int_R^{\bar{w}} \frac{1 - F(w)}{r + \delta + \lambda_1 [1 - F(w)]} \, dw \end{aligned}$$

- What happens as $\lambda_1 \to \lambda_0$?

Equilibrium

- Assume $0 \leq b < p < \infty$ and $0 < \lambda_i < \infty$ for $i = 0, 1$.

  1. No firm pays less than $R \Rightarrow R \geq \underline{\mathrm{w}}$
  2. No pass points: if there exists a mass point at $\tilde{w} < p$ then a firm can increase its wage to $\tilde{w} + \varepsilon \Rightarrow \ell(\cdot)$ would increase a lot (all the workers at the mass point) and profit per worker decrease only slightly.

- So $F(w)$ is continuous with compact support $[\underline{\mathrm{w}}, \bar{w}]$

Equilibrium

- The lower bound of $F(w)$

$$\ell(\underline{w}|R, F) = \frac{\delta m \lambda_0}{(\delta + \lambda_1)(\delta + \lambda_0)} \text{ for all } \underline{w} > R$$

Since this is a constant w.r.t. w we have that $\underline{w} = R$.

- In the support of $F(w)$ all profits are equal

$$\frac{(p - R)\delta m \lambda_0}{(\delta + \lambda_1)(\delta + \lambda_0)} = (p - w)\ell(w|R, F)$$

$$F(w) = \frac{\delta + \lambda_1}{\lambda_1}\left[1 - \left(\frac{p - w}{p - R}\right)^{\frac{1}{2}}\right]$$

- The upper bound of $F(w)$ is found with $F(\bar{w}) = 1$

Let's look at some data

- In the model the offer distribution $F(w)$ is different from the observed wage distribution $G(w)$.
  - $G(w)$ stochastically dominates $F(w)$
- Can we see this in wage data?
  - Christensen et al. (2001) look at Danish wage data
  - Calculate g as the observed wage distribution
  - Calculate f as the wage distribution of individuals hired out of unemployment

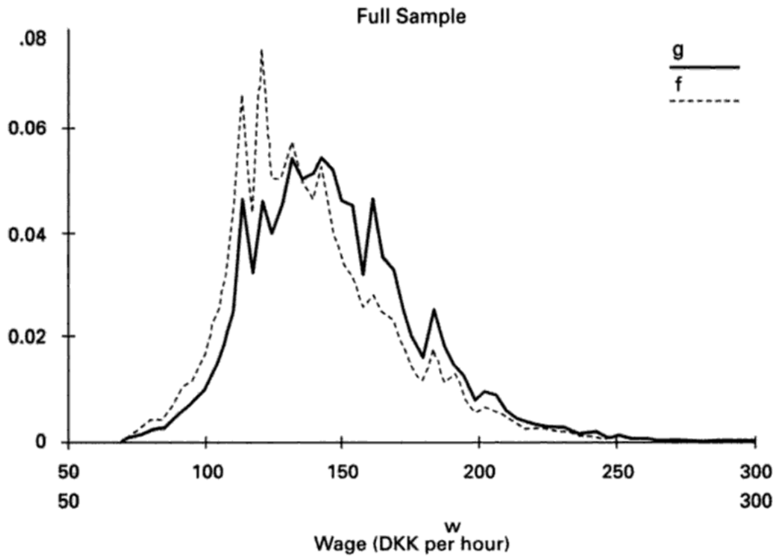

Figure 3.2
Offer (f) and wage (g) densities.

Some Critiques about Burdett-Mortensen

1. Why don't incumbent firms react to offers from outside firms trying to hire their workers?
   - Postel-Vinay and Robin (2002): allow for Bertrand competition between firms
2. All wage growth is generated from job-to-job movements. No wage growth within the same job.
   - Burdett-Coles (2003): allow firms to post wage-tenure contracts

For next time

- What is the job finding probability?
    - what does it depend on?
    - does it change over the business cycle?