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An Equilibrium Model of Search Unemployment

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This paper develops a simple general equilibrium model with sequential search in which a nondegenerate wage offer distribution is endogenously determined. We use this model to analyze the comparative statics effects of increases in unemployment compensation on the unemployment rate and aggregate welfare taking into account the induced change in the wage offer distribution. Our results differ significantly from the predictions of the standard "partial-partial" model. For example, one can expect a selective increase in unemployment compensation, made available to those who impute a relatively low value to leisure, to decrease the equilibrium rate of unemployment.

I. Introduction

This paper develops a simple equilibrium model of search unemployment. By this term we mean the unemployment resulting from the rational rejection of available wage offers by unemployed job seekers in favor of further search for more lucrative offers. We focus on the effects of unemployment compensation on the equilibrium rate of search unemployment, both because unemployment compensation is an important policy issue and because the standard analysis of unem-

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ployment compensation provides a convenient straw man against which to motivate our approach.

According to the standard search-theoretic model, an increase in unemployment compensation lowers the net cost of search to the unemployed, resulting in an increase in reservation wages and a consequent increase in the expected duration of search.[1] The key assumption underlying this model is that the wage offer distribution from which individuals search is exogenously given. This assumption is important for two reasons. First, we have no assurance that the idea of sequential search from a nondegenerate wage offer distribution makes any sense in equilibrium. There exists no simple general equilibrium model in which optimizing wage offers by firms combined with optimizing sequential search strategies by individuals result in a nondegenerate equilibrium wage offer distribution.[2] Second, even if one were to presume the existence of a nondegenerate equilibrium wage offer distribution, one would expect an increase in unemployment compensation to change that distribution. Since the standard comparative statics analysis of the effects of unemployment compensation is based on the notion of an exogenous and unchanging wage offer distribution, that analysis would seem to be of limited relevance (cf. Rosen 1977).

In this paper we develop a simple steady-state general equilibrium model with sequential search in which a nondegenerate wage offer distribution is endogenously determined. The search unemployment associated with the wage offer distribution is a Nash equilibrium outcome in the sense of being generated by the simultaneous optimizing behavior of firms and individuals in the economy. The equilibrium wage offer distribution and hence the equilibrium unemployment rate will vary with the amount of unemployment compensation available. We can therefore carry out a comparative statics analysis of the effects of unemployment compensation taking into account the endogeneity of the wage offer distribution.

[1] The model underlying this result is presented, e.g., in Lippman and McCall (1976*a*, 1976*b*). An example of the considerable empirical work that seems to support the predictions of the theory is Ehrenberg and Oaxaca (1976). The effects of imperfect "experience rating" in the system of financing unemployment compensation on the temporary layoff policies of firms have been stressed by Feldstein (1976) and are emphasized in the survey paper by Topel and Welch (1980). We do not deal with experience-rating issues in this paper.

[2] This is the well-known Rothschild (1973) criticism of the "partial-partial" nature of search theory. There are several models in the literature that feature nondegenerate endogenous wage (or price) offer distributions (e.g., Salop and Stiglitz 1977). However, almost all of these models are based on nonsequential, "noisy," or purely ad hoc search strategies; i.e., "dispersion equilibrium" is attained by sacrificing the tenet of sequential search. Three partial equilibrium models based on the sequential search strategy that generate dispersion equilibria are Axell (1977), Reinganum (1979), and Burdett and Mortensen (1980).

The basic ideas underlying our model are simple. We consider an economy in which a single good is produced with labor as the sole factor of production. Time is accounted in discrete periods, and in any period there are k individuals and n firms in the economy. Firms exist in perpetuity, but individuals suffer a constant "death risk" of τ $(0 < \tau < 1)$ in the sense that an individual will exit the economy with probability τ at the end of any period. There is thus a flow of τk individuals into and out of the economy per period.

The product market is assumed to be an "auctioneer market" characterized by perfect information. Accordingly, all firms must offer the product for sale at a common price, which we normalize to unity. The labor market, on the other hand, is assumed to be a "search market" characterized by imperfect information in the standard search-theoretic sense of individuals knowing the form of the wage offer distribution but not knowing (prior to search) the identity of firms making particular offers.

The simplest way to generate "dispersion equilibria" in a model of this type is to allow for heterogeneity among individuals and/or firms. Our key assumption is that there are two types of individuals in the economy differing according to the value "imputed" to leisure. This assumption ensures that there can be at most two wages offered in equilibrium.

Assume for the moment the existence of the two-wage dispersion equilibrium. Let w_0 and w_1 denote the low and high wage, respectively, and let γ denote the fraction of firms offering the low wage. In equilibrium w_0 must be the reservation wage of those individuals who impute a low value to leisure, and w_1 must be the reservation wage of those who impute a high value to leisure. Individuals who place a low value on leisure will accept the first wage offer encountered, whereas those who place a high value on leisure will search until they encounter w_1. The amount of search in the economy—the unemployment rate—is therefore an increasing function of γ.

The requirement that w_0 and w_1 be reservation wages leads to two equilibrium conditions relating w_0, w_1, and γ. The third equilibrium condition is provided by the requirement that each firm make a profit-maximizing choice between w_0 and w_1. That some firms prefer to offer w_0 while others prefer to offer w_1 is ensured by allowing for heterogeneity among firms. Specifically, we assume a continuum of firms differing according to a "productivity index."[3]

[3] An alternative way to close the model is to assume homogeneous firms and impose an "equal profits" condition, i.e., to require that all firms be indifferent between offering w_0 and w_1. However, existence of the two-wage dispersion equilibrium becomes tenuous with this approach.

In the sections that follow we first specify the decision problems faced by utility-maximizing individuals and profit-maximizing firms, allowing us to derive the three equilibrium conditions relating w_0, w_1, and γ (Secs. II and III). In Section IV we establish a simple sufficient condition for the existence of the two-wage dispersion equilibrium, and in Section V we examine the comparative statics of increases in unemployment compensation. The effects of an increase in unemployment compensation will in general depend upon the distribution of the productivity index; however, for a broad class of distribution functions we show that a general increase in unemployment compensation increases the equilibrium unemployment rate, even when the endogeneity of the wage offer distribution is taken into account. However, a selective increase in unemployment compensation, given only to individuals who place a low value on leisure, will, for the same class of distributions, decrease the equilibrium unemployment rate. In Section VI we examine the efficiency implications of unemployment compensation. In our model, increases in unemployment compensation can enhance welfare by reallocating workers to more productive firms, even in the face of increasing unemployment, and we present a simple example to illustrate this effect. Finally, in Section VII we offer a concluding discussion.

II. Individuals

An individual entering the economy in any period is assumed to follow an optimal sequential search strategy with the objective of maximizing expected lifetime utility. His utility in any period is assumed to be of the form $u = x + vm$, that is, the sum of utilities from consumption (x) and leisure (m). The parameter v imputes a "consumption value" to leisure.

The variable m takes on the value of zero or one according to whether the individual is working or searching (not working). The variable x is given by the sum of consumptions out of wage and nonwage incomes. If the individual is working at a wage of w, then his wage income is w; otherwise his wage income is zero. Nonwage income consists of "dividends," θ, that is, the individual's per period share in economy-wide profits plus any unemployment compensation, b, received. Unemployment compensation is financed by lump-sum taxation out of dividends. Dividends are received whether the individual is working or searching,[4] whereas unemployment compen-

[4] Note that dividends therefore do not enter the individual's decision calculus. We take advantage of this to assume that unemployment compensation is financed out of dividends. We are therefore able to abstract from any effects brought about by the system of financing unemployment compensation.

sation is received only during periods of search. Individual utility in any period is thus given by

$$u = \begin{cases} w + \theta & \text{if working at a wage of } w \\ \theta + b + v & \text{if searching.} \end{cases} \tag{1}$$

The individual's search problem is as follows. When he enters the economy, he draws a wage of w at random from the wage offer distribution. If he accepts w, then he starts work immediately, supplying one unit of labor and consuming $w + \theta$ per period, and he continues to work at that wage for the duration of his lifetime. If, on the other hand, he rejects w, then he "consumes" an imputed leisure of v, receives a nonwage income of $\theta + b$, and with probability $1 - \tau$ survives to draw another wage at random from the wage offer distribution in the next period. Note that since an individual's lifetime is a "memoryless" random variable—that is, the "death risk" is constant—an individual who survives to draw another wage faces a decision problem identical to the one faced on entering the economy.

We assume two classes of individuals differing according to "leisure values." Let β denote the fraction of individuals with low values of leisure, v_0, and let $1 - \beta$ denote the fraction with high values of leisure, v_1. All individuals draw from the two-point wage offer distribution, drawing a wage of w_0 with probability γ and a wage of w_1 with probability $1 - \gamma$.

In order for (w_0, w_1, γ) to be an equilibrium distribution, w_0 must be the reservation wage of the v_0 individuals and w_1 must be the reservation wage of the v_1 individuals. If w_0 were less than the reservation wage of the v_0 individuals, then firms offering w_0 could attract no workers. If, on the other hand, w_0 were to exceed the reservation wage of the v_0 individuals, then any firm offering w_0 could reduce its wage offer without suffering any loss in labor supply. Likewise, to attract any v_1 individuals, w_1 must be no less than the reservation wage of that group. On the other hand, were w_1 to exceed the v_1 individuals' reservation wage, then w_1 could be reduced without any loss in labor supply.

These facts allow us to derive two equilibrium conditions relating w_0, w_1, and γ. Consider an individual with value of leisure v_0 who has drawn a wage of w_0. If he rejects w_0, then he enjoys a period of leisure valued at v_0, receives a nonwage income of $\theta + b$, and with probability $1 - \tau$ survives to draw another wage. The wage sampled on the subsequent draw equals w_1 with probability $1 - \gamma$; and if w_1 is in fact drawn, then that wage is accepted, leading to an expected future lifetime utility of $(w_1 + \theta)/\tau$. Otherwise the search process continues.

Thus, the value of further search, that is, the value of rejecting w_0, is

$$V^* = v_0 + b + \theta + (1 - \tau)\left[\frac{(1 - \gamma)(w_1 + \theta)}{\tau} + \gamma V^*\right],$$

or

$$V^* = \frac{v_0 + b}{1 - \gamma(1 - \tau)} + \frac{(1 - \gamma)(1 - \tau)}{1 - \gamma(1 - \tau)}\frac{w_1}{\tau} + \frac{\theta}{\tau}.$$

But if w_0 is the reservation wage for v_0 individuals, then $V^* = (w_0 + \theta)/\tau$, the value of accepting w_0. Thus we have

$$w_0 = \frac{(1 - \gamma)(1 - \tau)}{1 - \gamma(1 - \tau)} w_1 + \frac{\tau(v_0 + b)}{1 - \gamma(1 - \tau)}. \tag{2}$$

The reservation wage property of w_1 is even simpler. The value of rejecting w_1 equals $(v_1 + b + \theta)/\tau$, whereas the value of accepting w_1 is $(w_1 + \theta)/\tau$. Thus,

$$w_1 - v_1 - b = 0.\text{[5]} \tag{3}$$

III. Firms

Firms are assumed to produce according to the linear production functions $y = \lambda l$, where the "productivity index" (output/worker) λ is distributed across firms according to the distribution function $A(\lambda)$.[6] As a normalization, we take the support of λ to be [0, 1]. Let $l(w)$ denote the per period labor supply to a firm elicited by a wage offer of w. Then the profits of a firm with productivity index λ as a function of its wage offer w are simply $\Pi(w; \lambda) = (\lambda - w)l(w)$.

In a dispersion equilibrium, a fraction γ of all "active" firms offers w_0 and a fraction $1 - \gamma$ offers w_1. (The concept of an active firm will be defined below.) The requirement that firms' wage offers be profit maximizing gives the final equilibrium condition relating w_0, w_1, and γ. The derivation of this equilibrium condition is illustrated in figure 1. First, firms with $\lambda \leq w_0$ do not operate; hence, only a fraction $1 - A(w_0)$ of all firms is "active." We assume that individuals search only

[5] In partial equilibrium search models a common necessary condition for the existence of a nondegenerate wage (or price) distribution is that the distribution of "search costs" not be bounded away from zero (see Axell 1977). In the general equilibrium context the forgone wage component of search cost is endogenously determined so that this necessary condition is automatically met.

[6] We will assume that λ has a differentiable density function, $a(\lambda)$, whenever it is convenient to do so.

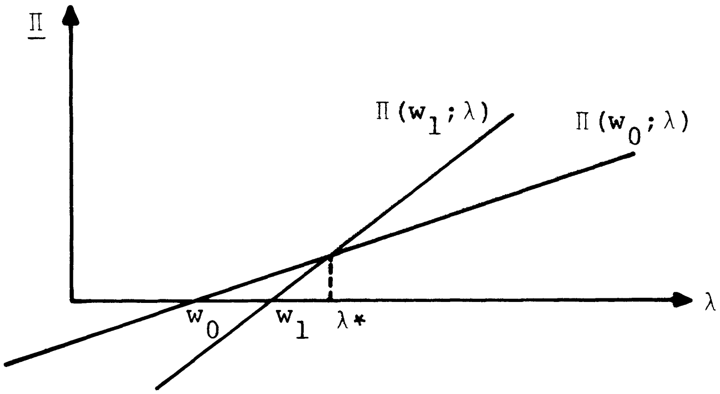

FIG. 1.—$\Pi(w_0; \lambda)$ and $\Pi(w_1; \lambda)$

from active firms. Next, let λ^* be defined by $\Pi(w_0; \lambda^*) = \Pi(w_1; \lambda^*)$; that is, λ^* is the productivity index such that a firm is indifferent between offering w_0 and w_1. Firms with $w_0 < \lambda \leq \lambda^*$ will offer w_0, while firms with $\lambda^* < \lambda \leq 1$ will offer w_1. Hence we have the equilibrium condition

$$\gamma = \frac{A(\lambda^*) - A(w_0)}{1 - A(w_0)}, \tag{4}$$

where

$$\lambda^* = \frac{w_1 l(w_1) - w_0 l(w_0)}{l(w_1) - l(w_0)}. \tag{5}$$

Finally, we need to derive $l(w_0)$ and $l(w_1)$. Consider a firm offering w_0. Only individuals with the low value of leisure, v_0, will accept this offer. There are $\tau k \beta\, v_0$ individuals entering the economy per period, and if we let $\mu \equiv k/n[1 - A(w_0)]$, the ratio of individuals to active firms, then there are $\tau\mu\beta$ such individuals per active firm entering the economy each period. All of the $\tau\mu\beta\, v_0$ individuals contacting a firm offering w_0 will accept that offer; hence, $l(w_0)$ can be computed as the sum of the $\tau\mu\beta$ individuals who accept w_0 in the current period, the $(1 - \tau)\tau\mu\beta$ surviving individuals who accepted w_0 in the previous period, and so on; that is,

$$l(w_0) = \tau\mu\beta[1 + (1 - \tau) + (1 - \tau)^2 + \ldots],$$

or

$$l(w_0) = \mu\beta. \tag{6}$$

Next, consider a firm offering w_1. All individuals contacting this firm accept w_1, and the number of contacts per firm per period is the sum of the

$\tau\mu\beta$	v_0 individuals entering the economy
$\tau\mu(1 - \beta)$	v_1 individuals entering the economy
$\tau\mu(1 - \beta)\gamma(1 - \tau)$	v_1 individuals who have searched once
$\tau\mu(1 - \beta)\gamma^2(1 - \tau)^2$	v_1 individuals who have searched twice
$\vdots$	

Thus, the number of acceptances per period is

$$\tau\mu\beta + \tau\mu(1 - \beta)[1 + \gamma(1 - \tau) + \gamma^2(1 - \tau)^2 + \ldots]$$

$$= \tau\mu\beta + \frac{\tau\mu(1 - \beta)}{1 - \gamma(1 - \tau)},$$

implying a labor supply of

$$l(w_1) = \mu\beta + \frac{\mu(1 - \beta)}{1 - \gamma(1 - \tau)}. \tag{7}$$

Note that we have derived the equilibrium unemployment rate (i.e., the fraction of individuals searching in any period) in passing. In any period there are $\tau k(1 - \beta)\gamma$ individuals who will search for the first time, $\tau k(1 - \beta)\gamma^2(1 - \tau)$ individuals who will search for the second time, and so on: the equilibrium unemployment rate is given by

$$s = \tau(1 - \beta)\gamma[1 + \gamma(1 - \tau) + \gamma^2(1 - \tau)^2 + \ldots]$$

or

$$s = \frac{\tau(1 - \beta)\gamma}{1 - \gamma(1 - \tau)}. \tag{8}$$

Since $ds/d\gamma = \tau(1 - \beta)/[1 - \gamma(1 - \tau)]^2$, the equilibrium unemployment rate is an increasing function of γ, as required.

IV. Equilibrium

The discussion above has established conditions that necessarily must hold given the existence of a two-wage dispersion equilibrium. Before using these conditions to investigate the properties of a dispersion equilibrium, we need to examine conditions *sufficient* for existence in terms of the exogenously given parameters of the model in order to ensure that the concept of the dispersion equilibrium is not vacuous.

It is useful to refer back to figure 1 to see what is involved. What needs to be ensured is that some firms have the incentive to "outbid" other, less productive firms.[7] Since we have normalized the support of λ to be [0, 1], this is equivalent to ensuring $0 < \lambda^* < 1$. The cutoff productivity λ^* is easily expressible in terms of the exogenous parameters of the model. From (5) we have

$$\lambda^* = \frac{w_1 l(w_1) - w_0 l(w_0)}{l(w_1) - l(w_0)} = w_1 + \frac{(w_1 - w_0) l(w_0)}{l(w_1) - l(w_0)}$$

$$= w_1 + \frac{(w_1 - w_0)\beta}{(1 - \beta)/[1 - \gamma(1 - \tau)]},$$

using (6) and (7). But $w_1 - w_0 = \tau(v_1 - v_0)/[1 - \gamma(1 - \tau)]$, using (2) and (3); hence

$$\lambda^* = v_1 + b + \frac{\tau(v_1 - v_0)\beta}{1 - \beta}. \tag{9}$$

The condition $0 < \lambda^* < 1$ is thus easily satisfied by a wide range of plausible choices for τ, β, v_0, and v_1.

To see that a "wide range of plausible choices" does indeed exist, it is useful to consider the alternatives. If $\lambda^* \leqslant 0$, then necessarily $v_1 + b < 0$; that is, even given the existence of unemployment compensation, all workers find the prospect of leisure so loathsome that they would instead be willing to work at a negative wage. In this case the equilibrium outcome will be full employment at the universal (negative) wage of $v_1 + b$. The case of $\lambda^* \geqslant 1$ comes about if the value placed on leisure by the v_1 individuals and/or the level of unemployment compensation is high enough to induce v_1 individuals to reject all wage offers any firm could profitably offer. In this case (given also that $v_0 + b \leqslant 1$) the equilibrium outcome will be a single wage of $v_0 + b$ together with an equilibrium unemployment (or nonparticipation) rate of $1 - \beta$.

[7] Since unemployment compensation is financed by lump-sum taxation out of dividends (see n. 4 above), we need also to ensure that total unemployment compensation payments do not exhaust dividends. That is, on a per capita basis we require that $sb \leqslant \theta$. For a fixed distribution of productivities this implies an upper limit on b. Alternatively, this upper limit can be changed by changing the assumed distribution of productivities. In the analysis below this condition is always assumed to be satisfied.

V. Comparative Statics

We are now in a position to examine how the equilibrium wage distribution (i.e., w_0, w_1, and γ) varies with the level of unemployment compensation.

PROPOSITION 1: A general increase in unemployment compensation has the following effects on the equilibrium wage distribution:

$$\frac{d\gamma}{db} = \frac{[1 - \gamma(1 - \tau)][a(\lambda^*) - (1 - \gamma)a(w_0)]}{\Delta}, \tag{10a}$$

$$\frac{dw_0}{db} = \frac{[1 - A(w_0)][1 - \gamma(1 - \tau)] - a(\lambda^*)(1 - \tau)(w_1 - w_0)}{\Delta}, \tag{10b}$$

$$\frac{dw_1}{db} = 1, \tag{10c}$$

where $\Delta = [1 - A(w_0)][1 - \gamma(1 - \tau)] - a(w_0)(1 - \gamma)(1 - \tau)(w_1 - w_0)$.

PROOF: Rewrite the equilibrium conditions as

$$[1 - \gamma(1 - \tau)]w_0 - (1 - \gamma)(1 - \tau)(v_1 + b) - \tau(v_0 + b) = 0$$
$$A(\lambda^*) - A(w_0) - \gamma[1 - A(w_0)] = 0,$$

regarding γ, w_0, and λ^* as implicit functions of b. Recall from (9) that $d\lambda^*/db = 1$. Differentiating the equilibrium conditions with respect to b then gives the following pair of equations:

$$\begin{bmatrix} (1 - \tau)(v_1 + b - w_0) & [1 - \gamma(1 - \tau)] \\ [1 - A(w_0)] & (1 - \gamma)a(w_0) \end{bmatrix} \begin{bmatrix} d\gamma/db \\ dw_0/db \end{bmatrix} = \begin{bmatrix} 1 - \gamma(1 - \tau) \\ a(\lambda^*) \end{bmatrix},$$

the solution to which is given by (10a) and (10b). Finally $dw_1/db = 1$ follows directly from (3).

The expressions above are rather formidable and seem to suggest that anything is possible depending on the distribution of the productivity index. However, it is possible to derive interesting qualitative results for a broad class of distribution functions.

PROPOSITION 2: Suppose $a'(\lambda^*) \geqslant 0$, that is, the density function of the productivity index is nondecreasing. Then a general increase in unemployment compensation (i) leads to an increase in the equilibrium rate of unemployment, that is, $d\gamma/db > 0$, and (ii) leads to a "widening" of the wage distribution, that is, $dw_0/db < dw_1/db = 1$.

PROOF: (i) If $a'(\lambda) \geqslant 0$, then $a(\lambda^*) \geqslant a(w_0)$, implying that the numerator of (10a) is positive. To show that the denominator is unambiguously positive, use

$$A(w_1) = A(w_0) + a(w_0)(w_1 - w_0) + \frac{a'(\tilde{w})}{2}(w_1 - w_0)^2$$

for some $\tilde{w}$ between w_0 and w_1. Then rewrite the denominator as

$$\Delta = [1 - \gamma(1 - \tau)][1 - A(w_0) - a(w_0)(w_1 - w_0)] + \tau a(w_0)(w_1 - w_0)$$

$$= [1 - \gamma(1 - \tau)][1 - A(w_1) + \frac{a'(\tilde{w})}{2}(w_1 - w_0)^2]$$

$$+ \tau a(w_0)(w_1 - w_0),$$

which is positive since $1 - A(w_1) > 0$ and $a'(\tilde{w}) \geqslant 0$. (ii) The result for dw_0/db follows from $a(\lambda^*) > (1 - \gamma)a(w_0)$.

PROPOSITION 3: Suppose again that $a'(\lambda) \geqslant 0$. Then a selective increase in unemployment compensation granted only to those with a low imputed value of leisure leads to a decrease in the equilibrium unemployment rate.

PROOF: The selective increase in unemployment compensation implies

$$\frac{d\lambda^*}{db} = -\frac{\tau\beta}{1 - \beta}.$$

Differentiating the equilibrium conditions with respect to b then yields

$$\begin{bmatrix} (1 - \tau)(v_1 + b - w_0) & 1 - \gamma(1 - \tau) \\ 1 - A(w_0) & a(w_0)(1 - \gamma) \end{bmatrix} \begin{bmatrix} d\gamma/db \\ dw_0/db \end{bmatrix}$$

$$= \begin{bmatrix} \tau \\ -a(\lambda^*)[\tau\beta/(1 - \beta)] \end{bmatrix}$$

or

$$\frac{d\gamma}{db} = \frac{-\tau a(w_0)(1 - \gamma) - [1 - \gamma(1 - \tau)]a(\lambda^*)[\tau\beta/(1 - \beta)]}{\Delta} < 0,$$

where $\Delta > 0$ is as given in the proof of the second proposition.

The intuition behind these results is not difficult. A general increase in unemployment compensation has the direct effect of increasing the high wage offer since the reservation wage of those who place the highest value on leisure varies directly with b. The effect on the low wage offer is less clear-cut. On the one hand, an increase in unemployment compensation increases the per period utility value of search. On the other hand, if γ increases, the probability that search will pay off decreases. Thus, so long as $d\gamma/db > 0$, the low wage offer will increase by less than the high wage offer. The presumption that γ will increase follows from the necessary increase in λ^*, the cutoff productivity. That is, an increase in unemployment compensation necessarily implies that the *number* of firms offering w_1 must decrease.

The condition that the density function of λ be nondecreasing ensures that the decrease in the number of firms offering w_1 is not more than offset by any decrease in the number of firms offering w_0.

However, if the increase in unemployment compensation is directed solely toward those with a low imputed value of leisure, then w_1 is unaffected. Individuals with the low leisure value begin to search more aggressively and the cutoff productivity λ^* falls. Again, the condition that the density function of λ be nondecreasing ensures that a decrease in λ^* translates to a decrease in γ.

VI. Efficiency

We have established that a general increase in unemployment compensation leads to increased unemployment for a broad class of distribution functions of productivity. One should not, however, be tempted to use this result to conclude that unemployment compensation is inefficient, that is, that the socially optimal level of unemployment compensation is zero.

Suppose the social objective function is per capita utility, $u^* = x^* + v_1 s$, that is, per capita consumption plus the value imputed to per capita leisure. To derive equilibrium per capita production (= equilibrium per capita consumption), first compute total production as the sum of production from low-wage firms and high-wage firms. Total production from firms offering w_0 may be computed as the product of three terms—(i) the number of firms offering w_0 ($= n[A(\lambda^*) - A(w_0)]$), (ii) $l(w_0)$, and (iii) the average productivity of firms offering w_0 ($= \int_{w_0}^{\lambda^*} \lambda dA(\lambda)/[A(\lambda^*) - A(w_0)]$). That is, total production from low-wage firms is simply $nl(w_0)\int_{w_0}^{\lambda^*} \lambda dA(\lambda)$, and, analogously, total production from high-wage firms is $nl(w_1)\int_{\lambda^*}^{1} \lambda dA(\lambda)$. Hence, equilibrium per capita consumption is

$$x^* = \frac{n}{k}\left[l(w_0)\int_{w_0}^{\lambda^*} \lambda dA(\lambda) + l(w_1)\int_{\lambda^*}^{1} \lambda dA(\lambda)\right]$$

$$= \frac{1}{\mu[1 - A(w_0)]}\left\{l(w_0)\int_{w_0}^{1} \lambda dA(\lambda) + [l(w_1) - l(w_0)]\int_{\lambda^*}^{1} \lambda dA(\lambda)\right\},$$

or

$$x^* = \beta\int_{w_0}^{1} \frac{\lambda dA(\lambda)}{1 - A(w_0)} + \frac{(1 - \beta)(1 - \gamma)}{1 - \gamma(1 - \tau)}\int_{\lambda^*}^{1} \frac{\lambda dA(\lambda)}{1 - A(\lambda^*)}. \quad (11)$$

Let $s_1 = \tau\gamma/[1 - \gamma(1 - \tau)]$, that is, the unemployment rate among the v_1 individuals. Then

$$x^* = \beta \int_{w_0}^{1} \frac{\lambda dA(\lambda)}{1 - A(w_0)} + (1 - \beta)(1 - s_1) \int_{\lambda^*}^{1} \frac{\lambda dA(\lambda)}{1 - A(\lambda^*)},$$

so that per capita utility is given by

$$u^* = \beta \int_{w_0}^{1} \frac{\lambda dA(\lambda)}{1 - A(w_0)} + (1 - \beta) \int_{\lambda^*}^{1} \frac{\lambda dA(\lambda)}{1 - A(\lambda^*)} - s \int_{\lambda^*}^{1} \frac{(\lambda - v_1) dA(\lambda)}{1 - A(\lambda^*)}. \tag{12}$$

The first two terms in (12) give "full-employment output," that is, the hypothetical level of per capita consumption that would be attained with full employment, and the last term gives the utility loss from unemployment.

The temptation to conclude that unemployment compensation must be socially inefficient results from the fact that so long as $ds/db \geq 0$, the utility loss from unemployment is increasing in b. The obvious point is that this temptation founders on the fact that full-employment output also depends on b. An increase in unemployment compensation causes the wage distribution to change in such a way that those who search (the v_1 individuals) are induced to seek out more productive firms. Likewise, if $dw_0/db > 0$, those who do not search will also become employed by more productive firms. The change in the wage distribution drives the least efficient firms out of the market.[8]

The trade-off between these two effects depends on the distribution function of λ. To provide some illustration we examine the simplest possible example, namely, $A(\lambda) = \lambda$, $0 \leq \lambda \leq 1$. The uniform distribution is particularly simple because it allows one to use the equilibrium conditions to find γ (or w_0) as the solution to a simple second-order equation. Once one solves for γ and w_0, the computation of s and u^* is straightforward.

Table 1 presents the equilibrium variables as functions of the level of unemployment compensation for selected values of τ and β, taking $v_0 = 0$ and $v_1 = 0.25$. Starting with $b = 0$, we compute $w_0(b)$, $w_1(b)$, $\gamma(b)$, $s(b)$, and $u^*(b)$ for increments of 0.05 in b up to the point where $\lambda^* \geq 1$. These examples suggest that the optimal level of unemploy-

[8] Unemployment compensation thus enhances efficiency by improving the match between workers and firms. This is not, however, a matching model in the standard sense since all individuals are equally productive at any given firm. For a matching model giving an efficiency analysis of unemployment compensation, see Diamond (1981).

TABLE 1

EXAMPLE BASED ON UNIFORM DISTRIBUTION OF PRODUCTIVITIES ($v_0 = 0$, $v_1 = .25$, Equilibrium Values of w_0, w_1, γ, s, u^* as Functions of b)

	$b = 0$	$b = .05$	$b = .10$	$b = .15$	$b = .20$	$b = .25$	$b = .30$	$b = .35$	$b = .40$	$b = .45$	$b = .50$	$b = .55$	$b = .60$	$b = .65$	$b = .70$
$\tau = .1, \beta = .1$:															
w_0	.22	.27	.32	.37	.42	.47	.52	.57	.62	.67	.72	.77	.82	.87	.90
w_1	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
γ	.037	.040	.043	.046	.050	.055	.061	.069	.079	.092	.110	.137	.182	.269	.509
s	.003	.004	.004	.004	.005	.005	.006	.007	.008	.009	.011	.014	.020	.032	.084
u^*	.624	.648	.673	.698	.723	.747	.772	.796	.821	.844	.868	.891	.912	.927	.913
$\tau = .1, \beta = .9$:															
w_0	.21	.26	.31	.36	.41	.45	.50	.54	.58	.61	.59	...	...	...	...
w_1	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	...	...	...	...
γ	.332	.355	.382	.414	.451	.496	.550	.616	.701	.809	.939	...	$\lambda^* > 1$	...	...
s	.005	.005	.006	.007	.008	.009	.011	.014	.019	.030	.061	...	...	...	...
u^*	.618	.642	.666	.690	.713	.735	.757	.777	.793	.799	.769	...	...	...	...
$\tau = .5, \beta = .1$:															
w_0	.11	.16	.21	.26	.31	.36	.41	.45	.50	.55	.59	.63	.67	.71	.73
w_1	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
γ	.170	.181	.193	.207	.224	.243	.266	.293	.327	.369	.422	.491	.583	.706	.867
s	.083	.089	.096	.104	.113	.125	.138	.155	.176	.203	.241	.293	.370	.491	.688
u^*	.593	.613	.633	.652	.670	.686	.701	.713	.721	.725	.721	.706	.670	.599	.467

ment compensation can be quite "high" relative to wages actually paid,[9] "despite" the fact that $s(b)$ is increasing in b, as indicated by our second proposition.

VII. Conclusion

In this paper we have developed a simple general equilibrium model of search unemployment and used the model to analyze the effects of unemployment compensation. The key feature of the model is the endogeneity of the wage offer distribution. We are thus able to (i) establish the logical consistency of sequential search as a general equilibrium phenomenon and (ii) analyze the comparative statics effects of increases in unemployment compensation taking into account the induced change in the wage offer distribution.

Our results on the effects of increasing unemployment compensation differ significantly from the corresponding effects predicted by the standard "partial-partial" model. Although one *can* expect a general increase in unemployment compensation to increase the equilibrium rate of unemployment, the direct incentive effect for individuals can be offset by firms' adjustment of the wage offer distribution to a considerable degree. More strikingly, one can expect a selective increase in unemployment compensation, made available to those who impute a relatively low value to leisure, to decrease the equilibrium rate of unemployment. This latter result is straightforward in our model but absurd under the standard approach.

Our results on the efficiency aspects of increasing unemployment compensation are also straightforward. The point is that increases in unemployment compensation bring about a reallocation of workers to more productive firms as a result of the change in the wage offer distribution. Of course this result is a direct consequence of the assumption about firm heterogeneity we made to close the model. However, it should be understood that the existence of an equilibrium wage dispersion seems to require assumptions that inevitably imply some sort of reallocation effect.

Although our model is of course stylized in many respects, it should be possible to weaken several of the key assumptions. Some obvious extensions would be (i) to assume more than two classes of individuals, (ii) to allow for a more general (e.g., nonadditive) utility function, and (iii) to consider alternative systems of financing unemployment compensation (e.g., by lump-sum taxation out of profits or by a uniform tax on wages). The main effect of these extensions seems to

[9] Of course, this is to some degree an artifact of our choices for v_0 and v_1 in the example.

be to complicate the arithmetic, eliminating the determinate, and thus didactic, flavor of the results. However, the primary moral of this paper, the need to explicitly take into account the endogeneity of the wage offer distribution, undoubtedly emerges unscathed.

A much more difficult extension, and one that is beyond the scope of this paper, would be to relax the steady-state nature of the analysis to explicitly study the dynamics associated with a change in the level of unemployment compensation. The reason that dynamics would be so difficult in a model of this type is that the reactions of both sides of the market (i.e., both individuals and firms) to a perturbation to an initial steady-state equilibrium need to be modelled, with each agent's optimal reaction depending on the optimal reactions of all other agents. One can reasonably expect, however, that dynamic considerations would not change the basic thrust of the analysis since, as a referee has helpfully emphasized, there is no stock variable in the model that is affected by the path taken from one steady state to another.

Finally, we conclude with an appeal to the empirically minded not to reject equilibrium models of search unemployment as irrelevant theorizing. Although the predictions of the "partial-partial" search model seem to be regarded as firmly established in the empirical folklore, we remain doubtful. First, at least one group of econometricians (Atkinson et al. 1984) has suggested that existing clear-cut empirical results on the effects of unemployment compensation are to some extent an artifact of artful specification. Second, and more fundamental, most empirical work in this area seems to ask the wrong question. The typical cross-section or panel analysis addresses the question of whether individuals who receive relatively generous unemployment compensation search more than others who receive less generous compensation, holding other differences between individuals constant. But the relevant policy question is whether economies characterized by relatively more generous unemployment compensation have more search unemployment than economies with less generous compensation. Such a question requires an equilibrium answer.

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