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On The Efficiency of Matching and Related Models of Search and Unemployment

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This paper describes a simple framework for evaluating the allocative performance of economies characterized by trading frictions and unemployment. This framework integrates the normative results of earlier Diamond-Mortensen-Pissarides bilateral matching-bargaining models of trade coordination and price-setting, and consists of a set of general conditions for constrained Pareto efficient resource allocation that are applicable to conventional natural rate models. To illustrate, several conventional models of the labour market are reformulated as matching-bargaining problems and analyzed using this framework.

1. INTRODUCTION

Is the equilibrium rate of search unemployment efficient? This is clearly an important question and one that has generated a large body of research since the Friedman-Phelps notion of a natural rate was first introduced. Almost twenty years later, however, the basic issues remain largely unresolved: some economists firmly believe the natural rate is too high, others believe it is too low, and still others believe it is just right! From a theoretical perspective, the problem here is not that different models yield different results; this much is obvious. Rather, the problem *appears* to be that the models in question have very little structure in common and hence offer no obvious basis for a comparison of their results. This appearance is deceptive.

In this paper I propose a general framework for evaluating the allocative performance of economies with trading frictions and equilibrium unemployment, a framework that readily identifies the common allocative structure among equilibrium search models with unrelated microfoundations. This work represents an integration and extension of the recent bilateral matching-bargaining models of Diamond (1981, 1982*a*, 1984*a*), Mortensen (1982*a*, *b*) and Pissarides (1984*a*, *b*); in these models, recall, potential traders are brought together pairwise by a given stochastic matching technology and, once together, their terms of trade are determined instantaneously as the outcome of a bargaining process that uses a given surplus-sharing rule.[1]

1. The early matching-bargaining papers are by Diamond and Maskin (1979) and Mortensen (1978). In addition to the papers already cited, more recent work includes Diamond (1982*c*), Diamond and Yellen (1985), and Pissarides (1985*a*, *b*; 1987); the relevant surveys are by Diamond (1984*b*) and Mortensen (1986). In this context Howitt's (1985) model of transaction costs and externalities in thin markets as well as the empirical pieces by Lancaster (1979) and Nickell (1979) on unemployment duration should also be noted.

The paper is divided into two parts. The first part constructs a simple matching model whose normative results subsume all earlier ones in this area. Drawing on Diamond (1982*b*, *c*), Mortensen (1982*a*) and Pissarides (1984*a*, *b*), Section 2 describes a labour market where unemployed workers search for jobs, firms with vacancies recruit employees and where, prior to meeting, the output produced by any given worker-firm pair is uncertain. Section 3 then identifies the different types of market externalities that can result in this setting and derives conditions which the matching technology and surplus-sharing rule must satisfy for each one to be internalized.

Taken together, these conditions for constrained Pareto efficiency require that each agent's social contribution and private gain from participating in the matching process be equal. Earlier normative results represent special applications of these general conditions; to illustrate, Section 4 confirms this view of Diamond's (1981, 1982*a*, 1984*a*) well-known but seemingly very different models of barter and monetary exchange.

The second part of this paper shows that these efficiency conditions comprise a general and broadly applicable framework that can be used to analyze conventional natural rate models that lack an explicit matching-bargaining structure. A two-stage process is involved: (1) For any conventional market situation, find the matching technology and surplus-sharing rule which together replicate that market's equilibrium allocation and price (or price distribution). (2) Apply the earlier conditions for constrained efficiency to the derived technology and sharing rule to evaluate the underlying market equilibrium.

Section 5 undertakes this exercise with three very different models of the labour market: a classical auction market; a market with imperfect information, in which firms search for employees and equilibrium is characterized by a non-degenerate price distribution; and a market with perfect information, in which workers search for jobs and the probability of finding a job depends on the numbers of workers and firms in the market. Together, these examples nicely illustrate how the matching framework in question can be applied to conventional markets independent of the participants' transaction and information technologies, of the active players' identities and choice sets and even of the type of equilibrium employed. Indeed, this independence is an essential feature of any framework that purports to be relevant for models that lack a common micro-structure.

These examples also highlight an important difference between matching models, as a distinct class of natural rate models, and conventional search models. Though wages in matching-bargaining models are completely flexible, these wages have nonetheless been denuded of any allocative or signaling function: this is because matching takes place before bargaining and so search effectively precedes wage-setting. This observation is true of matching models that appeal to cooperative bargaining theory to model wage negotiations, of the Diamond–Mortensen–Pissarides type, as well as those that use non-cooperative bargaining theory (Binmore and Herrero (1988), Gale (1987) and Rubinstein and Wolinsky (1985)).

In conventional market situations, by contrast, firms design their wage offers in competition with other firms to profitably attract employees; that is, wage-setting occurs prior to search so that firms' offers can influence workers' search behaviour and, in this way, firms' offers can influence the allocation of resources in the market. In other words, the matching technologies which bring trading partners together in matching models must be specified exogenously, whereas agents' search strategies which collectively perform the same function in conventional situations are endogenous and determined jointly with wages in equilibrium. Hence only in the latter situation can we ever expect the resulting equilibrium wage to potentially internalize players' search externalities.

Section 6 offers some final remarks.

2. A BILATERAL MATCHING MODEL

This section describes a steady-state labour market equilibrium in which unemployed workers search for jobs, firms with vacancies recruit employees and where, prior to meeting, the net product of any given worker-firm match is uncertain.

Let n and k respectively denote the numbers of risk-neutral works and firms in the market: these are fixed but large numbers. Assuming that each firm can employ only one worker, k also denotes the total number of jobs in the market. Letting u and v respectively denote the numbers of unemployed workers and vacant jobs, it follows that

$$n-u=k-v, \tag{1}$$

as the number of employed workers equals the number of filled jobs.

We assume that agents' common discount rate equals zero. This simplifies the analysis by allowing us to later directly compare steady-state solutions rather than having to determine the discounted value of the change in some variable along a convergent path from one solution to another; otherwise, this assumption is inessential.

Matching

An unemployed worker may spend s on search in any period, and a vacant firm may correspondingly spend r on recruitment. Unattached agents make contact with at most one potential trading partner per period. Let $p(s)=P(s;\hat{s},\hat{r},u,v)$, $p_s>0$, denote the probability that a worker who spends s on search will find (or be found by) a vacant job at the end of the current period, given that each of the remaining $u-1$ unemployed workers is spending $\hat{s}$ on search and that each of the v vacant firms is spending $\hat{r}$ on recruitment. Similarly, taking the symmetric actions of all other unattached agents as given, let $q(r)=Q(r;\hat{s},\hat{r},u,v)$, $q_r>0$, denote the probability that a firm which spends r on recruiting will find (or be found by) an unemployed worker at the end of the current period.

Let $m(s,r,u,v)$ denote the total number of worker-job contracts which result per period, in a symmetrical situation, when u unemployed workers each spend s on search and v vacant firms each spend r on recruitment. This aggregate matching technology has positive first derivatives in all of its arguments, and is linked to individual agents' contact probabilities by the following identities:

$$m(s,r,u,v)=uP(s;s,r,u,v)=vQ(r;s,r,u,v). \tag{2}$$

Our characterization of functions $p(s)$, $q(r)$ and $m(s,r,u,v)$ will otherwise remain intentionally vague.

Separation

At the end of each period a randomly selected fraction b of currently employed workers $n-u$ lose their jobs.

Production

A given worker and firm together produce the same amount of non-storable output y each period. Nevertheless, some worker-firm pairs are more productive than others; that is, output per period for any randomly chosen worker and firm is described by a distribution with continuous c.d.f. $F(y)$. Of course, not all such drawings need be mutually beneficial.

Setting the price of output to one, suppose that only those pairings which generate $y \geqq y^*$ per period are acceptable to both parties. In other words, an unemployed worker and vacant firm who make contact and draw $y < y^*$ will disengage and continue to search and recruit the following period. Let $a(y^*) = 1 - F(y^*)$ denote the probability that a randomly chosen worker-firm pair will produce $y \geqq y^*$. Since $a(y^*)$ also describes the fraction of all worker-firm contacts which is acceptable, a symmetric steady-state equilibrium must satisfy

$$a(y^*)m(s, r, u, v) = (n - u)b. \tag{3}$$

That is, in equilibrium, the number of unemployed workers who find acceptable jobs each period, $a(y^*)m(s, r, u, v)$, equals the number of employed workers who become unemployed, $(n - u)b$, and therefore total employment remains constant.

Search and recruitment

To start, let $w(y)$ denote the wage paid to a worker whose match generates y units of output, and let $\bar{w} = E(w(y) | y \geqq y^*)$ and $\bar{y} = E(y | y \geqq y^*)$ respectively denote the mean wage and mean output of an acceptable match. Additionally, suppose unemployed workers enjoy leisure z while vacant firms bear cost c per period.

With current expenditures $\{s, r\}$, an unemployed worker and vacant firm will respectively find an acceptable trading partner with probabilities $ap(s)$ and $aq(r)$ where $a = a(y^*)$. Thus, taking the actions of all other unattached agents as given, it can be shown that their respective maximal steady-state income flows are[2]

$$Y_u = z - s + ap(s)[\bar{w} - Y_u]/b, \tag{4a}$$

$$Y_v = -c - r + aq(r)[\bar{y} - \bar{w} - Y_v]/b, \tag{4b}$$

2. To derive this result in the case of workers, let $W_u(s)$ denote the expected present value of lifetime utility for an unemployed who expends s on search during every period of unemployment, and let $W_c(\bar{y})$ denote the corresponding expected PDV of a lifetime utility of an employed worker whose output is $\bar{y}$. Thus, using the discount rate δ, we have

$$W_e(\bar{y}) = [\bar{w} + bW_u(s) + (1 - b)W_e(\bar{y})]/(1 + \delta),$$

which gives

$$W_e(\bar{y}) - W_u(s) = [\bar{w} - \delta W_u(s)]/(\delta + b). \tag{a}$$

In turn, the expected PDV of lifetime utility of an unemployed worker who expends $\hat{s}$ on search this period and s in all future periods equals

$$W_u(\hat{s}) = [z - \hat{s} + ap(\hat{s})W_e(\bar{y}) + (1 - ap(\hat{s}))W_u(s)]/(1 + \delta).$$

Substituting from (a), this gives

$$(1 + \delta)W_u(\hat{s}) = z - \hat{s} + ap(\hat{s})[\bar{w} - \delta W_u(s)]/(\delta + b) + W_u(s). \tag{b}$$

Therefore, the optimal stationary search intensity satisfies

$$0 = -1 + ap_s(s)[\bar{w} - \delta W_u(s)]/(\delta + b). \tag{c}$$

Finally, defining the maximal steady-state income flow of an unemployed worker to be $Y_u = \delta W_u(s)$, and taking limits in (b) and (c) as δ goes to zero, we have

$$Y_u = z - s + ap(s)(\bar{w} - Y_u)/b$$

where s satisfies

$$\frac{\partial Y_u}{\partial s} = -1 + ap_s(\bar{w} - Y_u)/b = 0.$$

where s and r satisfy (5) below. Y_u represents the permanent (average) income of an unemployed worker and is equal to the sum of a flow term, $z-s$, plus an expected "capital gain" due to a change in employment status; the term $(\bar{w}-Y_u)/b$ represents the total expected surplus provided to workers from employment, and is found by applying the job-death rate b to the expected stream of returns from acceptable employment, $w(y)-Y_u$ for $y \geqq y^*$; and analogous interpretation from firms' perspectives is given to Y_v.

In a symmetrical Nash search-recruitment equilibrium, $\{s, r\}$ must satisfy the first-order conditions (see footnote 2)

$$\frac{\partial Y_u}{\partial s} = -1 + ap_s[\bar{w} - Y_u]/b = 0, \tag{5a}$$

$$\frac{\partial Y_v}{\partial r} = -1 + aq_r[\bar{y} - \bar{w} - Y_v]/b = 0, \tag{5b}$$

where $p_s = P_1(s; s, r, u, v)$, $q_r = Q_1(r; s, r, u, v)$ and, from (4),

$$Y_u = [b(z-s) + ap\bar{w}]/(b+ap), \tag{6a}$$

$$Y_v = [-b(c+r) + aq(\bar{y} - \bar{w})]/(b+aq). \tag{6b}$$

Wages

The wage rate $w(y)$ is agreed to after a worker and firm make contact and determine that their joint output exceeds y^*. We assume that this wage is given by

$$w(y) = Y_u + \theta(y - Y_u - Y_v),$$

so that each employed worker receives his reservation wage Y_u plus a fraction θ of his match-specific net product.

Two comments are in order. First, following earlier Diamond–Mortensen–Pissarides formulations, one could view this wage as the outcome of worker-firm negotiations that are modeled by the generalized Nash bargaining solution,

$$w(y) = \text{argmax } (w - Y_u)^{\theta}(y - w - Y_v)^{1-\theta},$$

where θ measures labour's "bargaining power"; none of the results that follow, however, depend on this or any other particular rationalization for $w(y)$. Second, θ need not be constant; one could let workers' surplus-share be some function of any off-the-job market variables, say $\theta = \theta(s, r, u, v)$.

Job acceptance

A match with productivity y is mutually acceptable if and only if $w(y) \geqq Y_u$ or $y \geqq Y_u + Y_v = y^*$. To further describe this reservation productivity, we need to first determine the mean acceptable wage $\bar{w}$. The conditional expectation of $w(y)$ is $\bar{w} = Y_u + \theta(\bar{y} - Y_u - Y_v)$: substituting for Y_u and Y_v from (6), and solving for $\bar{w}$ we get

$$\bar{w} = (z-s) + \theta(b+ap)S(\bar{y}), \tag{7a}$$

$$S(\bar{y}) = \frac{\bar{y} + (c+r) - (z-s)}{b + ap\theta + aq(1-\theta)}. \tag{7b}$$

Substituting (7) into (6) gives

$$Y_u = (z-s) + ap\theta S(\bar{y}), \tag{8a}$$

$$Y_v = -(c+r) + aq(1-\theta)S(\bar{y}). \tag{8b}$$

Notice, (7a) and (8a) imply that $(\bar{w} - Y_u)/b = \theta S(\bar{y})$, so that

$$S(\bar{y}) = (\bar{y} - Y_u - Y_v)/b$$

is the total expected net surplus from an acceptable match. Finally, $y^* = Y_u + Y_v$ and (8) gives

$$y^* = \frac{b[(z-s)-(c+r)] + [ap\theta + aq(1-\theta)]\bar{y}}{[b + ap\theta + aq(1-\theta)]} \tag{9}$$

which can be solved for y^* as $\bar{y} = E(y \mid y \geqq y^*)$.

Equilibrium

The equilibrium numbers of unemployed workers and vacant firms, their respective search and recruiting expenditures, and the mutually acceptable reservation productivity, $\{u, v, s, r, y^*\}$, are found by solving (1), (3), (9) and, from (5),

$$\frac{\partial Y_u}{\partial s} = -1 + ap_s\theta S(\bar{y}) = 0 = -1 + aq_r(1-\theta)S(\bar{y}) = \frac{\partial Y_v}{\partial r}. \tag{10}$$

3. LOCAL EFFICIENCY

A symmetric equilibrium is said to be constrained Pareto efficient whenever the corresponding allocation maximizes steady-state welfare. In this section we derive a set of necessary conditions which the sharing-rule and matching technology, θ and $m(\cdot, \cdot)$, must satisfy in order for the resulting equilibrium to be efficient.

Let Y denote the steady-state flow of aggregate utility and let $Y_e(y)$ and $Y_f(y)$ respectively denote the steady-state utilities of an employed worker and a filled firm who jointly produce y. Since $n-u$ employed workers each enjoy utility $Y_e(\bar{y})$, on average, and $k-v$ filled firms each enjoy utility $Y_f(\bar{y})$, on average, we have

$$Y = (n-u)Y_e(\bar{y}) + uY_u + (k-v)Y_f(\bar{y}) + vY_v,$$

which, in turn, simplifies to[3]

$$Y = (n-u)\bar{y} + u(z-s) - v(c+r).$$

3. $Y_e(y) = \delta W_e(y)$ and $Y_u = \delta W_u$ are the steady-state utilities of an employed worker who produces y and of an unemployed worker, respectively, where

$$W_e(y) = [w + bW_u + (1-b)W_e(y)]/(1+\delta),$$

$$W_u = [z - s + apW_e(\bar{y}) + (1-ap)W_u]/(1+\delta),$$

and $p = m/u$ is the probability of finding a match. Therefore,

$$Y_e(\bar{y}) = \delta W_e(\bar{y}) = \bar{w} + b(W_u - W_e(\bar{y})),$$

$$Y_u = \delta W_u = z - s + a(m/u)(W_e(\bar{y}) - W_u),$$

and, using $am = (n-u)b$, we have

$$(n-u)Y_e(\bar{y}) + uY_u = (n-u)\bar{w} + (z-s)u.$$

Similarly, it can be shown that

$$(n-u)Y_f(\bar{y}) + vY_v = (n-u)(\bar{y} - \bar{w}) - (c+r)v.$$

A planner will choose $\{s, r, y^*, u, v\}$ to maximize Y subject to constraints (1) and (3). The resulting Lagrangian is given by

$$L = Y + \lambda[a(y^*)m(s, r, u, v) - (n-u)b] + \mu(v - k - u + n)$$

and its multipliers, $\{\lambda, \mu\}$, are found by solving $\partial L/\partial u = \partial L/\partial v = 0$ for $\mu = (c+r) - \lambda a m_v$ and

$$\lambda = \frac{\bar{y} + (c+r) - (z-s)}{b + am_u + am_v}. \tag{11a}$$

Observe that λ represents the joint social marginal product of an additional employed worker-firm pair, whereas

$$\left.\frac{\partial L}{\partial n}\right|_{e=\bar{e}} = z - s + \lambda a m_u, \tag{11b}$$

$$\left.\frac{\partial L}{\partial k}\right|_{e=\bar{e}} = -c - r + \lambda a m_v, \tag{11c}$$

represent (for given levels of employment, $e = n - u = \bar{e}$) the separate contributions of an extra unemployed worker and an extra vacant firm, respectively: for example, an extra unemployed worker enjoys utility $z - s$ and increases the number of acceptable matches by am_u, so that total utility increases by $z - s + \lambda a m_u$. The remaining first-order conditions are:

$$\frac{\partial L}{\partial y^*} = mf(y^*)\left[\frac{\bar{y} - y^*}{b} - \lambda\right] = 0, \tag{11d}$$

$$\frac{\partial L}{\partial s} = -u + \lambda a m_s = 0, \tag{11e}$$

$$\frac{\partial L}{\partial r} = -v + \lambda a m_r = 0. \tag{11f}$$

Using the planner's solution, (11), the efficiency properties of the decentralized matching equilibrium are described below in terms of a series of possible entry/exit, job-acceptance and search/recruitment externalities. Together these sources of inefficiency exhaust those identified by the current matching literature; surprisingly, the sets of necessary conditions for efficiency in each case are almost identical.

Entry/exit externalities

While the supplies of capital and labour have as yet been held fixed, we are still able to determine whether their incentives for entry are efficient. Diamond (1982*b*) argues that this will generally not be the case as the presence of an additional worker (firm) makes it easier (harder) for vacancies to find workers but harder (easier) for workers to find jobs. Following Diamond, suppose our model represents one of many islands in an economy where unattached agents are freely mobile between islands and where their reservation utilities, Y_u and Y_v, are equated across all such islands in equilibrium. In this case, aggregate output is maximized and factor mobility is efficient only if unattached agents receive their social marginal products.

Therefore, the necessary conditions for efficient entry of labour and capital, for given levels of employment, are

$$\left.\frac{\partial L}{\partial n}\right|_{e=\bar{e}} = Y_u, \qquad \left.\frac{\partial L}{\partial k}\right|_{e=\bar{e}} = Y_v.$$

Substituting from (8) and (11b, c), these conditions respectively simply to:

$$\lambda m_u = p\theta S(\bar{y}), \tag{12a}$$

$$\lambda m_v = q(1-\theta)S(\bar{y}). \tag{12b}$$

In words, efficient entry requires that unattached agents of each type receive their social marginal product. In the case of labour, for example, an extra unemployed worker increases the number of acceptable matches (and hence increases the number of employed workers) by am_u, while the social contribution of an additional employed worker is λ; on the other hand, unemployed workers find acceptable trading partners with probability ap, and subsequently receive a fraction θ of the resulting (average) net surplus $S(\bar{y})$.

Substituting $p = m/u$, $q = m/v$ and the expressions for $S(\bar{y})$ and λ from (7b) and (11a), (12a) and (12b) respectively yield

$$m_u - \theta\frac{m}{u} = \frac{am}{buv}(\theta v m_v - (1-\theta)um_u), \tag{13a}$$

$$m_v - (1-\theta)\frac{m}{v} = \frac{am}{buv}((1-\theta)um_u - \theta v m_v). \tag{13b}$$

Therefore, whenever the matching technology exhibits constant returns to scale in unemployment and vacancies, i.e. whenever $m = um_u + vm_v$, (13a) implies that $m_u = \theta m/u$ is necessary for efficient labour mobility and (13b) implies that $m_v = (1-\theta)m/v$ is necessary for efficient capital mobility. More importantly, combining (13a, b), we see that labour and capital mobility are *jointly* efficient only if

$$m_u = \theta\frac{m}{u}, \qquad m_v = (1-\theta)\frac{m}{v}. \tag{14}$$

Hence matching technologies that exhibit constant returns to scale are necessary for efficient factor mobility; moreover, given constant returns, (14) defines the sharing-rule

$$\theta = um_u/m = \theta(s, r, u, v),$$

that internalizes all entry/exit externalities.

More on (12) *and* (14)

These conditions for efficient factor mobility have their parallel in the general theory of externalities. Since the latter theory is largely static, we can best illustrate this feature by first deriving the counterparts to (12) and (14) in the context of a much simpler one-period model.

Consider an economy with two sectors and fixed labour and capital endowments, $N = n_1 + n_2$ and $K = k_1 + k_2$. Sector 1 is conventional: it has a neoclassical production technology, $f(n_1, k_1)$, an output price equal to one, and has trade coordinated by a Walrasian auctioneer. Sector 2 is a simple version of our matching model: it has a Leontief technology, where one worker and one unit of capital together produce one unit output, an output price equal to ϕ, and a matching technology, $m(n_2, k_2)$, such that the n_2 workers (k_2 firms) who enter Sector 2 find a trading partner with probability m/n_2 (m/k_2).

In Sector 1, the rental prices of labour and capital respectively equal f_n and f_k; and in Sector 2, the expected incomes of labour and capital respectively equal $\omega m/n_2$ and $(\phi-\omega)m/k_2$, where ω is the expected wage agreed upon by each worker-firm pair (to simplify, each factor's reservation price is set to zero). The equilibrium allocation of resources in this 1-period model thus satisfies $f_n=\omega n/n_2$ and $f_k=(\phi-\omega)m/k_2$, whereas the optimal allocation maximizes $f(n_1,k_1)+\phi m(n_2,k_2)$ and hence satisfies $f_n=\phi m_n$ and $f_k=\phi m_k$. As a result, factor mobility is efficient only if the matching technology and expected wage in Sector 2 satisfy;

$$\phi m_n=\omega m/n, \qquad \phi m_k=(\phi-\omega)m/k, \tag{12'}$$

which is the static counterpart to (12).

The un-internalized externalities that result when (12′) fail are remedied in the standard manner (Varian (1984, pp. 259–263)). That is, when the social value of the marginal product of entry and the private expected return from entry differ, for one or both factors, factor-specific entry taxes or subsidies can be designed that induce the optimal allocation of resources.[4] In turn, the same basic policies can be implemented in the dynamic matching model of Section 2, allowing for the following difference between the static and dynamic frameworks.

In both models, the separate entry of an unattached worker and an unfilled firm into the matching process will affect the current matching probabilities of the other participants. These are the primary entry-induced externalities. By affecting the matching rate, however, the separate entry of either agent will also affect the exit rate of matched worker-firm pairs. The implications of joint exit differ by model.

In the one-period model, the exit of matched worker-firm pairs cannot affect the subsequent matching probabilities of those agents who remain unattached. In effect, the net surplus created by a match, ϕ, and the social contribution of an extra employed worker-firm pair, ϕ, are the exactly the same. The only externality is the primary entry-induced externality; defining $\theta=\omega/\phi$, (12′) is immediately equivalent to (14).

In our dynamic matching model, however, the exit of a matched worker-firm pair will affect the subsequent matching probabilities of those agents who remain behind. When these secondary entry-induced externalities are not internalized, the expected net surplus created by a match, $S(\bar{y})$, and the social contribution of an extra employed worker-firm pair, λ, will not be equal (these externalities are the subject of the next subsection). On the other hand, from (12) and (14), efficient factor mobility implies that $S(\bar{y})=\lambda$ and hence implies that both the primary and secondary entry-induced externalities are internalized.

The intuition underlying (14) is basically the same as the static intuition underlying (12′), since (14) can be rewritten as

$$(\bar{y}-Y_u-Y_v)m_u=(\bar{w}-Y_u)m/u, \qquad (\bar{y}-Y_u-Y_v)m_v=(\bar{y}-\bar{w}-Y_v)m/v.$$

Unlike (12′), however, the left-hand-sides of these equations do not generally measure of the social value of the marginal product of unattached labour and capital because of the secondary externalities; the only exceptions, where $\lambda=(\bar{y}-Y_u-Y_v)/b$, are when either factor mobility or job-acceptance is efficient.

4. Under a "missing markets" interpretation, these taxes and subsidies define the prices faced by a notional employment agency, with "production function" $m(\cdot,\cdot)$, that pays labour and capital to participate and in turn receives a payment for each match it creates (see Greenwald and Stiglitz (1988)).

Job-acceptance externalities

Workers and firms stop searching as soon as they contact an agent with whom they can produce at least y^* units of output. Pissarides (1984a) has argued that this stopping rule is generally inefficient because the acceptance of a match causes a worker and firm to exit, and this changes the job matching probabilities of existing searchers in ways that exiting agents ignore.[5]

Solving (11d) for the socially optimal value gives $y^{**} = y - \lambda b$; subsituting from (11a) then yields

$$y^{**} = \frac{b[(z-s)-(c+r)]+(am_u+am_v)\bar{y}}{(b+am_u+am_v)}.$$

Comparing y^{**} to y^* in (9), it follows that the equilibrium and optimal values will coincide whenever the sharing-rule and matching technology satisfy

$$m_u + m_v = \theta\frac{m}{u} + (1-\theta)\frac{m}{v}. \tag{15}$$

To interpret this result, observe that efficient factor mobility implies an efficient reservation productivity, as (14) implies (15). In other words, whenever the externalities caused by the independent entry of a worker and a firm are each internalized, it is not surprising that whatever externality results from their joint entry (following the rejection of a match) will also be internalized. On the other hand, we expect their joint decision to exit and produce to be efficient whenever the private and social contributions of an additional employed worker are the same. This is indeed the case; from (7b) and (11a), $S(\bar{y}) = \lambda$ implies (15) which implies $y^* = y^{**}$.

Search/recruitment externalities

Mortensen (1982a) argues that agents' search and recruitment expenditures are generally inefficient because agents optimal expenditure decisions ignore the share to be obtained by their future trading partners.[6] Equating the corresponding social and private marginal contributions of search and recruitment gives

$$\frac{\partial L}{\partial s} = u(\partial Y_u/\partial s), \qquad \frac{\partial L}{\partial r} = v(\partial Y_v/\partial r)$$

so that, from (10) and (11e, f), we have

$$\frac{\lambda m_s}{u} = p_s\theta S(\bar{y}), \qquad \frac{\lambda m_r}{v} = q_r(1-\theta)S(\bar{y}). \tag{16}$$

Since the marginal contact probabilities $\{p_s, q_r\}$ differ from $\{m_s/u, m_r/v\}$ only in that elements of the latter set account for the adjustment of *all* unattached agents' search and recruiting expenditures, it follows from our earlier interpretation of (12) that, whenever $S(\bar{y}) = \lambda$, equations (16) simply equate the marginal social and private benefits for matching of search and recruitment.

5. These externalities are also described by Diamond (1982*c*, 1984*a*, *b*) and Mortensen (1984). The concensus view is that y^* will likely be less than the socially optimal value, with the result that too many jobs are accepted and total unemployment is deficient.

6. These externalities are also described by Mortensen (1982*b*) and Pissarides (1984*b*).

Unfortunately, this is all we can say about efficient search and recruitment without imposing further restrictions on the model. As a result, we now adopt the following two-stage characterization in which search and recruitment conceptually precede matching.

A two-stage model of search

Consider the following two-stage process. Starting at the second stage, with $\hat{u}$ unemployed workers and $\hat{v}$ vacant firms, suppose the available matching technology generates $x(\hat{u}, \hat{v})$ worker-firm contacts. Working backwards to the first stage, suppose that an unemployed worker who spends s on search enters the second stage with probability $\sigma(s)$, and that a vacant firm which spends r on recruitment enters with probability $\rho(r)$. Thus, starting with u unemployed workers and v vacant firms, this 2-stage process gives[7]

$$p(s) = \sigma(s)m(\hat{s}, \hat{r}, u, v)/\sigma(\hat{s})u, \tag{17a}$$

$$q(r) = \rho(r)m(\hat{s}, \hat{r}, u, v)/\rho,u, v)/\rho(\hat{r})v, \tag{17b}$$

where

$$m(s, r, u, v) = x(\sigma(s)u, \rho(r)v). \tag{17c}$$

Now, with search and recruitment modeled as the means of entry into a matching process, we expect the associated externalities to resemble those previously attributed to labour and capital mobility.

From (17), we can derive $m_s = m_u\sigma_s u/\sigma$, $m_r = m_v\rho_r v/\rho$, $p_s = m\sigma_s/\sigma u$ and $q_r = m\rho_r/\rho v$. Then, substituting these expressions into (16) gives (12): therefore, (13a) is now necessary for efficient search, (13b) is necessary for efficient recruitment, and (14) are necessary for efficient search and recruitment. Observe that (14) is equivalent to[8]

$$x_1 = \frac{\theta x}{\sigma u}, \qquad x_2 = \frac{(1-\theta)x}{\rho v}. \tag{18}$$

Further results

In Appendix A we confirm that (18) are also required whether each recruiting firm has a fixed number of vacancies to fill, as above, or has a constant returns to scale technology and hence an unlimited number of positions. (Note that the asset value of an individual vacancy is zero in the latter case as firms rather than vacancies are the scarce resource.)

4. DIAMOND'S MODELS OF BARTER EXCHANGE

Recently, Diamond (1981, 1982*a*, 1984*a*) has developed a series of dynamic equilibrium matching models of barter and monetary exchange in which, at least metaphorically, economic activity consists of picking coconuts and searching for trading partners. This section shows that his welfare results are special cases (18).

7. This type of separability is a common property of matching models: see Diamond (1982*a*) and Mortensen (1982*a*).

8. When the matching technology $x(\sigma u, \rho v)$ exhibits constant returns to scale for all positive σu and ρv, the planning solution and the decentralized equilibrium will both be unique (see Diamond (1982*a*, 1984*a*, *b*)), in which case (18) is both necessary and sufficient for constrained efficiency. On the other hand, it is clear that a non-CRS matching technology is inconsistent with constrained efficiency; in particular, taking such a non-CRS technology as given, we see that neither a fixed sharing-rule of the Diamond–Mortensen–Pissarides type, nor some endogenous rule that is, say, the equilibrium outcome of strategic behaviour in a noncooperative bargaining model (cf. Rubinstein and Wolinsky (1985)), is able to internalize the resulting externalities.

Diamond (1982a) develops a model of barter exchange with identical risk-neutral individuals in which, at any time, each individual has either 0 or y units for sale. The former are looking for production opportunities: the probability of finding an opportunity is given and each generates y units of output and costs c units to produce, where y is fixed and $c \sim g(c)$. A cut-off cost c^* is chosen by individuals so that only those projects with $c \leqq c^*$ are undertaken. (A production opportunity is a coconut tree, and a randomly chosen tree has height c.) Production and consumption are instantaneous. However, individuals can neither consume their own output (they must trade it for others' output) nor can they produce with unsold goods on hand.

Individuals trying to sell output are referred to as employed. With e employed agents, the given matching process is such that each one contacts a trading partner with probability $b(e)$, $b' \geqq 0$, and so a total of $eb(e)$ matches result per period. Matched traders exchange inventories, consume, and subsequently look for further production opportunities.

Diamond shows that the resulting equilibrium is locally inefficient whenever $b' > 0$. Individuals in this model fail to internalize the collective effect of their cut-off choice c^* on the matching probability $b(e)$. In particular, as more traders make trade easier when $b' > 0$, and as the likelihood of becoming a trader is an increasing function of c^*, the equilibrium cut-off cost will be less than socially optimal.

To apply our earlier analysis, consider the following variation on Diamond's model. Suppose that individuals are either blue or green and that half the population is blue. Furthermore, suppose blue (green) individuals produce blue (green) output but derive utility only from consuming green (blue) output. Assuming that individuals are otherwise identical, the analysis goes through exactly as before except that the blue-green matching process involves $e/2$ traders of each type and generates

$$eb(e) = x(e/2, e/2)$$

matches.

Looking for a production opportunity and choosing c^* is analogous here to the search/recruiting processes modelled earlier by (17). The important efficiency-related properties of the barter model are that there are equal numbers of blue and green individuals and that their implicit sharing rule is symmetric. Thus, in terms of our earlier analysis, $n = k$ (hence $u = v$), $\alpha(\cdot) = \rho(\cdot)$, $z = c = 0$ and $\theta = 1/2$. It then follows from (18) that an efficient allocation will result only if the matching technology exhibits constant returns to scale and symmetry ($x_1 = x_2$) which, from the previous equation, delivers $b' = 0$, which is Diamond's efficiency condition.

The interpretation of Diamond (1981) in light of (18) is similar, and omitted; Appendix B establishes that (18) comprise the necessary conditions for local efficiency in Diamond (1984a) as well.

5. SOME CONVENTIONAL MARKET MODELS

This section illustrates how conditions (14) and (18) can be used as a general framework for analyzing efficiency issues in conventional market economies. In turn, we consider an auction market, Butters' (1977) advertising model of a non-degenerate price distribution, and an economy exhibiting search externalities, due to Peters (1984), that provides a strategic micro-foundation for the matching technologies in Hall (1979), Nickell (1979) and Pissarides (1985a). These three examples were chosen because their micro-specifications are entirely different and yet each can be reformulated as a matching problem and hence compared directly to the others within that framework. Throughout this section we set $z=c=0$ and $\sigma(\cdot)=\rho(\cdot)=1$ which implies $s=r=0$.

Example 1: *An auction market*

Consider a one-period model with I unemployed workers and J vacant firms. Suppose that an auctioneer publicly sets a surplus-sharing rule to equate demand and supply, that all factors are freely mobile, and that each firm can only employ one worker. Thus, if $I<J$, all I workers secure employment at the equilibrium surplus-share $\theta=1$, and $J-I$ of the original J firms remain vacant; if $I \geqq J$ all J firms hire a worker at the equilibrium surplus-share $\theta=0$, and $I-J$ of the original I workers remain unemployed.

To use this model as a micro-foundation for $m(\cdot,\cdot)$ and θ, we need only assume that the subset of unattached agents who fail to meet a trading partner in any period is chosen at random. In this case, the model in Section 2 goes through exactly as before except that, now, an unemployed worker's contact probability equals $p=\min[1, v/u]$; a vacant firm's contact probability equals $q=\min[u/v, 1]$; the resulting number of matches equals $m(u,v)=\min[u,v]$; and the equilibrium sharing-rule is

$$\theta=\begin{cases}1 & u<v,\\ 0 & u \geqq v.\end{cases}$$

Thus, defining $m_u(d,d)=0$, we have

$$m_u=1=\theta\frac{m}{u},\qquad m_v=0=(1-\theta)\frac{m}{v},\qquad u<v,$$

$$m_u=0=\theta\frac{m}{u},\qquad m_v=1=(1-\theta)\frac{m}{v},\qquad u\geqq v,$$

and, as a result, this equilibrium allocation always satisfies (14).[9]

Observe that auction market economies of this type are characterized by a piece-wise-linear CRS matching technology. Mortensen (1982*a*, *b*) has shown that an efficient allocation will result with a linear matching technology only if a system of property rights can be established which circumvents the bargaining problem and allocates the entire surplus associated with a match to the agent reponsible for its formation, i.e. the matchmaker. Implementing Mortensen's scheme may thus be difficult; it requires ex ante commitment by all players coupled with the ability to accurately and publicly identify the matchmaker among any pair of traders. Of course these are precisely the functions that are performed here by an auctioneer when the scarce factor in the market is identified as the relevant matchmaker.

Example 2: *Butters' equilibrium advertising model*

The efficiency of an auction market model of matching is certainly not surprising. More interestingly, this result is really only a special case of the following more general proposition: whenever workers fail to experience external effects in matching, that is, whenever the probability that a worker will contact a firm is independent of the numbers

9. If each firm has a constant returns to scale production technology, rather than only one vacancy, all unattached workers will secure employment at the competitive surplus-share, i.e. $m=u$ and $\theta=1$, and hence (14) are again satisfied.

of participating workers and firms, so that $m_u = m(u, v)/u =$ constant, an efficient allocation will result only if employed workers receive the entire surplus associated with acceptable matches.

To illustrate this idea with a non-Walrasian example, we now consider Butters' (1977) well-known one-period advertising model in which the equilibrium price distribution is non-degenerate. In particular, we will show that the aggregate equilibrium matching process corresponding to this model can be summarized by a very simple technology, $m(u, v) = \delta u$ where $\delta > 1$, and that workers' equilibrium surplus share is one; therefore, as established by Butters' via an entirely different route, the equilibrium volume of advertizing is efficient.

Suppose all jobs generate the same output y, so that $a = 1$, and each firm has a CRS technology. Starting with u workers and k firms suppose, as in Butters (1977), that workers are initially uninformed and passive, and that each firm can send job offers informing workers of their wage and location at a fixed cost t per offer. These offers are allocated randomly across workers and, among those received by any given worker, the highest one above Y_u is accepted. Since the asset value of a vacancy is zero when there are constant returns (so that $Y_v = 0$; see Appendix A), the expected profit accounted for by a single offer at wage w equals

$$\pi(w) = \frac{(y - w)}{b} q(w) - t \tag{19a}$$

where $q(w) = \exp(-R(w))$ is the probability that a worker receives no offer at a wage $\gamma \geqq w$ and $R(w)$ is the number of offers sent per worker by all firms at wages $\gamma \geqq w$.

Following Butters it can be shown that, in equilibrium, the expected (average = marginal) advertising profit at each wage satisfies $\pi(w) = 0$ for all $w \in [Y_u, y - bt]$; in turn, this allows us to solve for $q(w)$ and the equilibrium distribution $R(w)$ over the same range. In this equilibrium the expected number of workers hired at wages $\gamma \geqq Y_u$, divided by the total number of workers to whom offers are sent, equals

$$\frac{m}{u} = \int_{Y_u}^{y-bt} -q(w)\, dR(w) = 1 - \frac{bt}{y - Y_u}. \tag{19b}$$

Consider a matching model in which workers contact firms with probability m/u, draw a random output $y(\xi) \geqq Y_u$ and assign the surplus share θ to the worker where output per match is defined by $y(\xi) = y - bt/q(\xi)$ and $\xi \sim dR(\xi)$ (note that $bt/q(\xi)$ is the amortized expected cost per hire at wage ξ); this matching model is equivalent to an advertising model in which output is nonrandom but firms randomly send out wage offers from a distribution $R(w)$ such that the probability that a worker receives any offer is given by (19b).

With $Y_v = 0$, $w(\xi) = Y_u + \theta(y(\xi) - Y_u)$; hence (19a) implies $\theta = 1$. In addition, since

$$Y_u = (m/u)\theta S(\bar{y}) = (m/u)[(\bar{y} - Y_u)/b], \tag{19c}$$

we have that (19b) describes m/u as a function of Y_u, while (19c) describes Y_u as a function of m/u; solving we thus get $m/u = \delta =$ constant.

From (13a), and (13a′) in Appendix A, and due to the absence of external effects for unattached workers, it follows from $m_u = m/u$ that their decisions to enter a Butters-type matching process will be efficient only if employed workers receive the entire surplus from an acceptable match; and this in turn is a consequence of the fact that firms earn zero net expected profits, which is an equilibrium condition in Butters' model. Other models which are likewise summarized by $m = \delta u$ and $\theta = 1$ included Prescott (1975) and

Lucas and Prescott (1974): in the latter case, employed workers are paid their marginal product which, in a world with decreasing returns, is exactly the surplus created by a match.[10]

It should be noted that Wilde's (1977) search model also has a non-degenerate equilibrium price distribution and an equilibrium allocation that is captured by the simple technology $m = \delta u$, $\delta < 1$. Unlike the Butters' situation, however, the social contribution of an additional worker in this model exceeds the private gain as firms enjoy strictly positive profits. In fact, any search model where workers fail to experience external effects and where firms have monopsony power ($\theta < 1$), say, because search is costly and workers have imperfect information (Burdett and Judd (1983); MacMinn (1980)) or firms have bargaining power (Albrecht and Jovanovic (1986)) fall into the same group. In these cases, where there is no search externality to internalize, the underlying inefficiency is basically the standard imperfect competition inefficiency.

Example 3: Search externalities

The auction market model and Butters' advertising model are efficient but may be misleading; both models fail to exhibit the type of congestion effects that underlie the externalities identified earlier in Section 3. Due to the possibility of monopsonistic behaviour, we already know that the absence of external effects does not guarantee efficiency. In this subsection we will show that the presence of these external effects, on the other hand, is not synonomous with inefficiency. An economy is described below in which agents experience/generate external effects and yet all such effects are internalized by the equilibrium wage.

To start, consider a one-period model in which each vacant firm first announces a sharing-rule and, taking these offers as given, each unemployed worker then chooses a strategy for visiting these firms. To simplify notation, the expected net surplus from any worker-firm match is set to unity. We begin with I unemployed workers and J vacant firms: after describing the symmetric equilibrium for finite I and J, we allow these numbers to grow large holding $I/J = \mu$ fixed.

The available search technology is quite simple: a worker is able to visit only one firm and, if unable to make contact and secure employment there, he withdraws for the remaining period. A worker's search strategy specifies, for each vacant firm, the probability that he will visit that firm. Since each vacant firm has only one opening, we assume that one worker is chosen at random, and so the probability that a worker who visits a firm will also make contact with that firm equals $1/(1+i)$ when i other workers also visit that firm. In this way, any given worker's optimal search strategy will depend upon the strategies chosen by all other searching workers.

An equilibrium is established as follows. For $i = 1, \ldots, I$, the i-th worker chooses a search strategy to maximize expected utility given the remaining $I-1$ workers strategies

10. The Lucas-Prescott model employs a 2-stage matching process in which an *inter*-market allocation is followed by an *intra*-market allocation. Since the former process in their formulation depends on neither the number of participating workers nor their individual search intensities, it follows that the resulting natural rate will be constrained efficient only if the *intra*-market matching process itself satisfies (18). Earlier we showed that an auction market does indeed satisfy these conditions. However, we also described a market with imperfect information, costly search and wage-setting firms where equilibrium is characterized by a non-degenerate wage distribution and unemployment, but which satisfies (18). In this sense, the Walrasian auctioneer on each of the Lucas-Prescott "islands" is not really an essential part of the story. In particular, if an alternative non-CRS inter-market matching technology is used, such as described in Diamond (1982*a*), then neither an auction market nor any other island-specific matching model will be consistent with constrained efficiency.

and vacant firms' surplus-sharing rules: a workers' search equilibrium is a Nash equilibrium in search strategies. Then, for $j=1,\ldots,J$, the j-th firm chooses a sharing-rule to maximize expected profits given the remaining $J-1$ firms' offers and workers' equilibrium search strategies. A full equilibrium is a Bertrand–Nash equilibrium of the latter rule-setting game. Observe that this market is characterized by perfect information as all agents' strategies are publicly known.

Following Peters (1984), it can be shown that (i) the probability that a worker who visits firm j will also be chosen by that firm, when all other workers visit j with probability α, equals[11]

$$p(\alpha)=(1-(1-\alpha)^I)/\alpha I;$$

(ii) the probability that at least one worker will visit j equals

$$q(\alpha)=1-(1-\alpha)^I;$$

and (iii) there exists a symmetric equilibrium where all workers visit each firm with probability $\alpha=1/J$ and all firms offer the same surplus share θ satisfying[12]

$$\frac{1-\theta}{\theta}+\frac{J}{(J-1)}\left[1-\frac{(J-1)}{I}\frac{(1-(1-J^{-1})^I}{(1-J^{-1})^I}\right]=0.$$

Now, holding $I/J=\mu$ fixed and letting I become arbitrarily large, we use $\lim(1-(\mu/I))^I=e^{-\mu}$ to evaluate the above expressions and derive the following limiting equilibrium matching probabilities, for workers and firms,

$$p=\frac{1-e^{-\mu}}{\mu},\qquad q=1-e^{-\mu}, \tag{20a}$$

and workers' equilibrium surplus-share

$$\theta=\frac{\mu e^{-\mu}}{1-e^{-\mu}}. \tag{20b}$$

Notice, as the ratio μ of workers to firms approaches zero (infinity), the probability p that an unemployed worker will be chosen from among those who arrive at a firm

11. The probability that a worker who visits a firm will also make contact with that firm equals $1/(1+i)$ when i other workers also visit that firm. Therefore, the probability that a worker who visits a firm will make contact, when all other workers visit that firm with probability α, equals

$$\begin{aligned}\textstyle\sum_{i=0}^{I-1}\min\left[1,\frac{1}{1+i}\right]\frac{(I-1)!}{(I-1-i)!\,i!}\alpha^i(1-\alpha)^{I-1-i}\\ &=(1-\alpha)^{I-1}+\frac{1}{\alpha I}\textstyle\sum_{i=1}^{I-1}\frac{I!}{(I-1-i)!(1+i)!}\alpha^{i+1}(1-\alpha)^{I-1-i}\\ &=(1-\alpha)^{I-1}+\frac{1}{\alpha I}\textstyle\sum_{i=2}^{I}\frac{I!}{(I-i)!\,i!}\alpha^i(1-\alpha)^{I-1}\\ &=(1-\alpha)^{I-1}+\frac{1}{\alpha I}(1-(1-\alpha)^I-\alpha I(1-\alpha)^{I-1}).\end{aligned}$$

12. To derive this expression, observe that $\theta_j p(\alpha)$ is expected utility of a worker who visits firm j given that all other workers visit j with probability α; hence the equality, $\theta_j p(\alpha)=\theta p(\beta)$, guarantees that if the strategy of visiting j w.p. α and each of the remaining firms w.p. β is used by $I-1$ workers, then it is optimal for the I-th worker to use it as well. Therefore, taking θ elsewhere as given, firm j choses θ_j to solve

$$\max\,(1-\theta_j)q(\alpha)\text{ s.t. }\theta_j p(\alpha)=\theta p(\beta),$$

where $\beta=(1-\alpha)/(J-1)$. Substituting $\theta_j=\theta$ and $\alpha=1/J$ into the first-order conditions of this problem gives the expression in the text.

approaches one (zero), the probability q that a vacant firm will be visited by at least one worker approaches zero (one), and workers' surplus-share θ approaches one (zero).

Considering the labour market model described earlier in Section 2, suppose the process of matching unattached agents and of dividing their joint surplus, which takes place each period, is now modeled by the symmetric equilibrium outcome of the above share-setting market game. Therefore, (20a) implies that the probability an unemployed worker will contact a vacant firm equals

$$p = \frac{[1-\exp(-u/v)]}{u/v};$$

the probability a vacant firm will be visited by at least one unemployed worker equals

$$q = [1-\exp(-u/v)];$$

and so the resulting number of matches will equal

$$m(u, v) = v[1-\exp(-u/v)]. \tag{21a}$$

This example is of special interest for two reasons.

First, the externalities identified earlier are present here as well: that is, increasing the number of unemployed workers u decreases (increases) the probability that a given unemployed worker (vacant firm) will find a job (worker); and increasing the number of vacant firms v increases (decreases) the probability that a given unemployed worker (vacant firm) will find a job (worker). Second, the exact same matching function has been derived elsewhere, in the absence of strategic considerations, by simply imposing a specific matching process (Hall (1979), Pissarides (1985*b*)). For example, starting with u workers and v firms, suppose each worker visits one firm chosen at random, so that the probability that a given worker will visit a particular firm is $1/v$. As the numbers of participants grow large, holding $\mu = u/v$ fixed, the probability that a given firm will *not* be visited by any worker approaches $e^{-\mu}$, which allows us to directly derive p, q and $m(u, v)$ as above.

Of course, the essential difference between such mechanical processes and our share-setting game is that only in the latter case does wage-setting precede search in a manner that allows workers and firms to compete for jobs and employees. Wages are described by $w(y) = Y_u + \theta(y - Y_u - Y_v)$, but now, from (20b), the equilibrium sharing rule is

$$\theta = \theta(u, v) = \frac{(u/v)\exp(-u/v)}{1-\exp(-u/v)}. \tag{21b}$$

As a result, workers' surplus-share is an increasing function of the demand for labour (as measured by the number of vacant firms) and a decreasing function of supply (as measured by the number of unemployed workers).

The matching function and sharing-rule described by (21) satisfy (13)–(15). As a result, the corresponding steady-state allocation must involve efficient capital and labour mobility, efficient job-acceptance and efficient search and recruitment.

While this particular matching function can, as we have seen, be derived in alternative settings, the equilibrium sharing-rule in (21b) is unquestionably unique to the wage-setting game described above; and the important property of this particular search game for normative analysis, despite the rationing of both workers and jobs in equilibrium, appears to be that all players's wage and search strategies are publicly known. We know from an earlier example that neither perfect information nor full employment nor even a non-degenerate wage distribution are required for constrained efficiency; as a consequence, we now conjecture that perfect information is essential for constrained efficiency whenever workers and firms experience external effects.

6. CONCLUDING REMARKS

This paper studies the efficiency of the natural rate of (un)employment. Sections 2 and 3 derive general conditions for efficient resource allocation in economies with risk-neutral agents where trade coordination and price-setting are the outcomes of a given matching technology and bargaining process. These conditions are remarkably simple and yet exhaust those described in earlier matching models. Section 5 then applies this framework to evaluate the allocative performance of several well-known equilibrium search models; this is a straightforward exercise as all such models have matching-bargaining representations.

In conventional analyses, one determines an individual agent's optimal search strategy taking the actions of all other agents as given. In the case of firms, these actions include their wage offers and must be credible; if not, search strategies are indeterminate. For example, Section 5 describes a Butters-type model in which firms send wage offers to workers, and each worker accepts the highest offer he receives; if firms cannot be held to their offers, however, competition becomes impossible as wage offers are ephemeral and the model breaks down. With ex-ante wage commitment ruled out, wages can only be determined after workers and firms meet; wages are still endogenous, but the process that brings buyers and sellers together, the matching technology, is now outside the model.

Whether or not ex-ante wage commitment is possible in a large number of labour markets is an empirical question that bears on the positive contribution of the matching model as a distinct natural rate model. The view taken in this paper is that, both from positive and normative perspectives, the major contribution of the matching model is as a short-hand representation for conventional market equilibria. That is, while the many examples and counter-examples of efficient natural rate models cited here hardly ever present a common micro structure, their aggregate equilibrium properties can in each case be represented as the outcome of some matching-bargaining problem. As a result, the comparative allocative properties of models as diverse as Butters' advertising model and Diamond's coconut trading economies are discovered by asking whether the unattached agents who participate in the corresponding matching process receive more or less than their social marginal product. To the extent that this is an easily formulated question, as seems likely, the goal of this paper has been achieved.

APPENDIX A

Matching with constant returns in production

In this appendix we relax the assumption that each firm can employ only one worker and show that, with only minor qualifications, the results in Sections 2 and 3 still hold. Thus, suppose each firm has a constant returns to scale technology for producing output and can hire as many workers as it contacts in any period. The model in Section 2 then goes through exactly as before except for the following three features:

First, a symmetric steady-state equilibrium satisfies

$$a(y^*)m(s, r, u, k) = (n-u)b, \tag{3'}$$

where k firms each spend r on recruitment. Second, recruiting firms enjoy the maximal steady-state income flow given by

$$Y_k = -c - r + aq(r)(1-\theta)S(\bar{y}), \tag{8b'}$$

where c is the fixed cost of hiring and $q(r)$ is the expected number of workers contacted by a firm which spends r on recruitment. Third, as the asset value of an unfilled job is nil when each firm's total employment is unrestricted, $Y_v = 0$, the average surplus of an acceptable hire is $S(\bar{y}) = (\bar{y} - Y_u)/b$ and the minimally acceptable output per job is $y^* = Y_u$.

The corresponding planning problem is also straightforward: choose $\{u, s, r, y^*\}$ to maximize $L' = Y' + \lambda'[am - (n-u)b]$ where $Y' = (n-u)\bar{y} + u(z-s) - k(c+r)$. Proceeding as in Section 3, it can be shown that the necessary conditions for efficient entry decisions by workers and firms respectively yield

$$m_u - \theta\frac{m}{u} = 0, \tag{13a'}$$

$$m_k - (1-\theta)\frac{m}{k} = \frac{am}{buk}((1-\theta)um_u - \theta km_k), \tag{13b'}$$

which together imply that $m_k = (1-\theta)m/k$; that $y^{**} = y^*$ is equivalent to (13a'); and that the conditions for efficient search and recruitment, given $m = x(\sigma u, \rho k)$, are simply $x_1 = \theta x/\sigma u$ and $x_s = (1-\theta)x/\rho k$.

From an efficiency perspective, the difference between this model and the one in Section 2 comes down to the difference between (13) and (13'). When each firm can employ only one worker, an additional unemployed worker makes it easier for jobs to find workers and harder for workers to find jobs. When each firm's total employment is unrestricted, an additional unemployed worker still makes it easier for firms to find workers but can have no effect on the employment prospects of the other unemployed workers. This accounts for the difference between (13a) and (13a'). On the other hand, (13b) and (13b') are identical because, whether or not employment per firm is restricted, an additional firm makes it easier for workers to find jobs and harder for other firms to find employees.

APPENDIX B

Diamond's model of monetary exchange

Diamond's (1984a) model of monetary exchange involves 3 different agents: n_m individuals try to exchange money for consumption goods, n_g individuals try to exchange inventory goods for money, and the remaining $u = n - n_m - n_g$ individuals, having neither money nor goods, look for production opportunities to acquire inventories (to sell for money, to buy someone else's goods for consumption, etc.). The population n is fixed, individuals are identical and risk-neutral, consumption and production are instantaneous, and production opportunities are found randomly which generate 1 indivisible unit of output and cost $c \sim g(c)$. Again, only those projects costing $c \leqq c^*$ are undertaken.

Each individual who finds an acceptable project enters on the supply-side of the trade process with goods in hand; he then shifts to the demand-side after selling his goods for money; and subsequently exits after exhausting his money balances to look for further production opportunities. This matching process is assumed to generate $f(n_g, n_m)$ completed transactions among n_g individuals with goods and n_m individuals with money. In steady-state equilibrium, the sale of one inventory unit exactly finances the purchase of one consumption unit.

Diamond argues that there are potential externalities associated with agents' willingness to produce, c^*, and with the supply of real balances. Let W denote the aggregate present discounted value of utility for the economy (the planner's objective function), let W_m denote an individual's equilibrium expected PDV of utility conditional upon having money to buy, and let W_u denote the corresponding term conditional upon having neither money nor inventories. The necessary conditions for local efficiency are then:

$$\frac{\partial W}{\partial c^*} = 0, \qquad \left.\frac{\partial W}{\partial n_m}\right|_{\dot{n}_g = 0} = W_m - W_u.$$

Observe that the latter condition equates the social and private gains from additional real balances; the private gain is the increase in lifetime utility in shifting from having neither money nor inventories to having money. It can be shown that these equations imply that the matching technology $f(n_g, n_m)$ must satisfy

$$f_1 = \theta f/n_g, \qquad f_2 = (1-\theta)f/n_m.$$

Diamonds employs the equal sharing rule, $\theta = 1/2$, but this feature is inessential.

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