# EC9A2 Problem Set 2

## 1 Comparative Statics with Depreciation Rate ($\delta$)

Consider the Ramsey model with exogenous labor supply (normalized to 1). The economy is characterized by:

- Preferences: $U = \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $\beta \in (0, 1)$
- Technology: $y_t = f(k_t)$ with $f'(k) > 0$, $f''(k) < 0$
- Capital accumulation: $k_{t+1} = (1 - \delta)k_t + i_t$
- Resource constraint: $c_t + i_t = f(k_t)$

(a) Derive the steady-state conditions for capital ($k^*$) and consumption ($c^*$) as functions of the model parameters. (You can start from the first oder conditions, you do not need to derive them.)

(b) Analyze the effect of an increase in the depreciation rate $\delta$ on the steady state using phase diagram analysis. Show algebraically and graphically how the $\Delta k = 0$ locus and $\Delta c = 0$ change when $\delta$ increases from $\delta_L$ to $\delta_H$. Illustrate the new steady state $(k^{**}, c^{**})$ compared to the original $(k^*, c^*)$ on your phase diagram.

(c) Assuming the economy is initially in steady state at $(k^*, c^*)$ when $\delta$ increases unexpectedly and permanently to $\delta_H$, describe and illustrate the transition dynamics. What happens to consumption on impact? How does the economy move to the new steady state?

(d) Why does higher depreciation affect the steady-state capital stock the way it does? Explain the mechanism through the Modified Golden Rule condition.

## 2 Equivalence of Planner's Problem and Competitive Equilibrium

Consider the Ramsey model with endogenous labor supply. The economy has:

- Preferences: $U = \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - \ell_t)$
- Technology: $Y_t = F(K_t, L_t)$ with constant returns to scale
- Capital accumulation: $K_{t+1} = (1 - \delta)K_t + I_t$
- Time endowment: $\ell_t \in [0, 1]$

(a) Given technology and the resource constraint, the social planner wants to maximize total utility. Write down the social planner's problem in discrete time. State the objective function the planner maximizes and write all the constraints. Clearly identify which variables are choice variables and which are state variables.

(b) Solve the social planner's problem using the Lagrangian method. Derive the first-order conditions and solve for the planners Euler equation and intratemporal optimality condition.

(c) Recall that the for the decentralized competitive equilibrium what we solved in lecture, the household optimality conditions are:

$$u_c(c_t, 1 - \ell_t) = \beta u_c(c_{t+1}, 1 - \ell_{t+1})[r_{t+1} + 1 - \delta]$$

$$\frac{u_{1-\ell}(c_t, 1 - \ell_t)}{u_c(c_t, 1 - \ell_t)} = w_t$$

When are these the same as the social planner's optimality conditions?

(d) Show that when $F(K, L)$ exhibits constant returns to scale, firm profits are zero in equilibrium, and verify that the resource constraints are equivalent in both problems.

(e) Explain why constant returns to scale is crucial for the equivalence between the social planner's problem and the decentralized problem. What role does zero profit play?