EC9A2 Problem Set 3

The Model

Consider the Ramsey model with exogenous labor supply (normalized to 1):

Preferences:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t) = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

where $\beta \in (0,1)$ is the discount factor and $\sigma > 0$ is the coefficient of relative risk aversion.

Technology:

$$y_t = f(k_t) = k_t^\alpha$$

where $\alpha \in (0,1)$ is the capital share.

Capital accumulation:

$$k_{t+1} = (1-\delta)k_t + i_t$$

where $\delta \in (0,1)$ is the depreciation rate.

Resource constraint:

$$c_t + i_t = f(k_t)$$

Combining the capital accumulation and resource constraint:

$$k_{t+1} = (1-\delta)k_t + f(k_t) - c_t$$

Bellman equation: (for the planner's problem, i.e. we are using the resource constraint)

$$V(k) = \max_{c,k'} \{u(c) + \beta V(k')\}$$

subject to:

$$\begin{aligned} k' &= (1-\delta)k + f(k) - c \\ c &\geq 0 \\ k' &\geq 0 \end{aligned}$$

Parameter Values:

- $\beta = 0.96$
- $\sigma = 2$
- $\alpha = 0.33$
- $\delta = 0.1$

1 Analytical Steady State

(a) Before implementing the numerical solution, derive the analytical steady state and calculate the value of steady state consumption, capital, and the capital output ratio, $k^*/f(k^*)$, at the given parameter values. (You can start from the equations you solved from in the PS2 Question 1a.)

(b) Interpret the Modified Golden Rule condition $f'(k^*) = \rho + \delta$ in the context of your numerical steady state. Calculate $f'(k^*)$ and verify it equals $\rho + \delta$ where $\rho = \frac{1-\beta}{\beta}$.

2 Setting Up Value Function Iteration

(a) Explain the value function iteration algorithm in your own words. Your explanation should include:

  (i) What the value function represents

  (ii) Why we need to discretize the state space

  (iii) The iterative process (how we update the value function)

  (iv) The convergence criterion

  (v) What the policy function tells us

(b) Propose a reasonable range for the capital grid: $k \in [k_{min}, k_{max}]$. Justify your choice.

(c) Propose a reasonable initial guess for $V_0(k)$. Explain why your choice might speed convergence.

3 Implementation in MATLAB

(a) Write a MATLAB script solves the model using VFI. The script should produce, the converged value function and the policy functions. Plot the value function $V(k)$. Plot the policy functions $g_c(k)$ and $g_k(k)$. On the plot of the capital policy function, include a 45 degree line.

(b) What is the steady-state capital from your policy function $g_k(k)$? (Find where $g_k(k) = k$). How close is this to the analytical k^* you calculated in Problem 1?

(c) What is the steady-state consumption from your policy function? How close is this to the analytical c^* you calculated in Problem 1?

(d) Looking at your graph for $g_k(k)$, when is the policy function above the 45 degree line and when is it below. What does this tell you about capital accumulation/decumulation?

(e) Starting from $k_0 = 0.5k^*$, use your policy function to simulate the path of capital and consumption for 50 periods. Plot the time paths of k_t and c_t. (HINT: you will need to use the matlab function "interp1" for this.)