EC9A2 Problem Set 4

Model Setup

Consider an infinitely-lived representative household that maximizes expected lifetime utility:

$$\max_{\{c_t, h_t, k_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

where the period utility function is:

$$u(c, h) = \frac{c^{1-\sigma}}{1-\sigma} - \chi \cdot h$$

The household faces the budget constraint:

$$c_t + k_{t+1} = f(k_t, h_t) + (1-\delta)k_t$$

The production function is:

$$f(k, h) = k^{\alpha} h^{1-\alpha}$$

Parameters:

- $h_t \in \{0, \bar{h}\}$ is the discrete labor choice (not work or work full-time)
- $\bar{h} = 0.4$ is the time endowment devoted to work when working
- $\chi = 0.5$ is the disutility of working (captures commuting costs, loss of leisure, etc.)
- $\beta = 0.96$
- $\sigma = 2$
- $\alpha = 0.33$
- $\delta = 0.1$

1 Theoretical Analysis

(a) Write the household's problem in recursive form using a value function $V(k)$. Be explicit about the discrete choice over $h \in \{0, \bar{h}\}$.

(b) Show that the value function can be written as:

$$V(k) = \max\{V^W(k), V^N(k)\}$$

where $V^W(k)$ is the value of working and $V^N(k)$ is the value of not working. Write out the Bellman equations for both $V^W(k)$ and $V^N(k)$ explicitly.

(c) For each labor choice $h \in \{0, \bar{h}\}$, derive the Euler equation relating optimal capital accumulation today to next period's capital. Use the envelope theorem to express $\frac{\partial V(k)}{\partial k}$ for each labor choice.

(d) Derive the condition under which the household chooses to work ($h = \bar{h}$) versus not work ($h = 0$). This should be an inequality involving $V^W(k)$ and $V^N(k)$.

Provide economic intuition: what factors make working more attractive? How does the capital stock affect this decision?

(e) Consider the steady state where $k_{t+1} = k_t = k^*$ and $c_{t+1} = c_t = c^*$.

- (i) Suppose the household works in steady state ($h^* = \bar{h}$). Derive the steady state capital-output ratio $k^*/f(k^*, h^*)$ as a function of the model parameters. How does it compare to the baseline Ramsey model without labor choice (you solve for this in PS3)?
- (ii) Now suppose the household does not work in steady state ($h^* = 0$). What is the steady state capital stock? Is this a stable steady state?
- (iii) Under what parameter conditions would we expect a working steady state versus a non-working steady state? Provide a condition involving χ, $\bar{h}$, and other parameters.

2 Computational Implementation

(a) What is the state space and control space? What do we need an initial guess for? In what order should you solve the two value functions $V^W(k)$, $V^N(k)$, and $V(k)$?

(b) Write a MATLAB script to solve the model using value function iteration. The script should produce, the converged value functions and the policy functions. Plot $V^W(k)$ and $V^N(k)$ on the same graph. Plot the optimal labor choice $g_h(k)$ as a function of k. Plot the capital policy function $g_k(k)$ along with the 45-degree line to identify steady states.

(c) At what value of k does the worker become indifferent between working and not working ($\bar{k}$)? Does what you find make sense? What is the value of the steady state that comes out of the VFI and how does it compare to the analytical value?